# Humans can run on water using big instantly-changable shoes 

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#### Abstract

Glasheen and McMahon correctly calculated that some small lizards should be able to run on water, large ones might have difficulty and people can't. However, Minetti et al have shown that humans probably could run on water in reduced gravity. And here we show that humans can also run on water, at least in the sense that Glasheen and McMahon calculated, if they have big shoes that they don't have to lift at each step.


## Introduction

This note is a follow up to the paper by Minetti at al where they showed that the prediction, by Gleeshan and McMahon, that humans cannot run on water might be circumvented in reduced gravity $[1,2]$. Here we show another circumvention: big shoes.

The nature of terrestrial locomotion. For horizontal locomotion near the surface of the earth there is need for both support, to keep from falling to the center of the earth, and propulsion to fight friction. Machines and animals use various strategies for these in air, on land and in and on water. Often noted are the parallels and contrasts between the continuous motions typically used for locomotion by machines, and the oscillatory motions typically used by larger animals. Airplanes use a steadily moving wing for support and steadily rotating propeller or turbine for propulsion. One the other hand animals and helicopters use the same wings for support as for propulsion, with a helicopter using steady rotation of its blades while birds and bats flap their wings. On the ground, most cars and bicycles both support and propel themselves with wheels on the ground while dinosaurs, frogs and people use reciprocating legs.

In and on the water both animals and watercraft most commonly use hydrostatics for support. For propulsion animals tend to use oscillatory motions of fins, feet or arms while most boats use propellers. But there are water vehicles that use hydrodynamic, rather than hydrostatic, support including hydrofoils and planing boats. These tend to use one mechanism for support (a planing surface or underwater wing) and a rotary propeller for locomotion. Using the same hydrodynamics for both support and propulsion, while less common in water than air, is used by dolphins using fin thrust for both support and propulsion when they swim short distances with most of their bodies out of the water. Can people support and propel themselves with flapping wings? If the wings are in the air, the question is still open. But it is possible if the wings are in the water. The oscillating wing craft, the Pogofoil, was one of the first to do this [3].

An odd example on water that uses the same surface for support and propulsion is a fast- moving snow-mobile. A fast snowmobile can skim on the water using the same surface for both support and propulsion ${ }^{1}$. Of course there are other mechanisms for locomotion including rowing, skating, rocket packs, traveling waves along the body (snakes), rocking a sailboat in dead wind, etc. In this paper we focus on a legged analogue to a snow-mobile on water; in this case, both the support and propulsion come from 'feet'.

Running on water. Given the equations governing hydrodynamic support (see Eqn. 1 in Methods), can we predict who can run on water? Gleeshan and McMahon found that small lizards should be able to, that large lizards should have trouble and that people are under-powered by a factor of 20 . Nonetheless, slapping water with feet does generate a vertical force and Minetti et al showed that running in place is possible if the subjects weight is reduced by about a factor of 7 (that is like $g<1.5 \mathrm{~m} / \mathrm{s}^{2}$ ) and if the feet are slightly enlarged (with small fins). So running on the water in reduced gravity should be possible also.

What if people had big water shoes? Following the calculation assumptions in Gleeshan et al for lizards, where they assumed that the foot retraction force could be neglected, what if we also made the foot retraction force-free. The idea comes from a comment after a McMahon seminar in about 1992: Nick Trefethen said "I've got it, Lily pads!" to which Tom McMahon said "That's the best comment anybody has ever made at a seminar I've given." Twenty years later we tried it. The idea is to have flat plates on the water that provide little direct buoyancy. The person then steps from plate to plate getting support

[^0]by the same forces as used in the lizard calculations.
Calculating what a person or animal is capable of, a priori, can be done only approximately at best. For example, no calculation yet reasonably predicts how fast people can run on land. But we do have some reasonable constraints. Equation 1 (see methods) predicts that the mechanical power needed for body support gets arbitrarily small as the frequency of stepping gets arbitrarily large. This is similar to the towing force on water skis decreasing as the speed increases. But people can't swing their legs arbitrarily fast. Gleeshan and McMahon assume that the effort of leg swing is great enough so that people can't take one step in less than 0.26 s . Consistent with that, the Minetti et al subjects chose a step time from 0.26 s to 0.31 s . For our theory we use the somewhat longer step period chosen by our subject, $t_{s}=0.4 s$. Our subject had a flight time of about 0.1 s so we further assume that the foot can press on the water for all of that $t_{c}=0.3 \mathrm{~s}$ (i.e., a duty factor of .375 for each foot). We calculated the power required using two force profiles: a constant descent rate and a constant force (see methods).

## Results

Pre-trials used three single plates with areas of $0.47,0.78$ and $1.58 \mathrm{~m}^{2}$. The subject could easily hop on and off the large plate, could barely hop on and off the middle plate, and sank with the small plate no matter how quickly he tried to step. So for our running trial we used ten $1 m^{2}$ plates. We abandoned the idea of wearing the plates as snow-shoe-like water shoes, they were just too big and awkward. Our subject was a healthy 39 year old male who started his runs at the corner of a swimming pool. In the first three trials with spacings of 1.6 m (center to center) he could not run without losing balance. With a spacing reduced 1.2 m he could, after a failed run or two, run the full length of 10 pads reliably, getting tired, but able to do the task several times in a 30 minute period (see Figure 1 and online movie). It turns out that balance is a key issue. The pads tip if the force if off center. In normal running foot placement is the main mechanism for balance. On our pads the person has to land near the center at every step and this largely removes the use of foot placement for balance. Thus an accurate centered start is needed, or else the subject ends up falling off to one side.

## Discussion

Although the Froude number of our experiments was about 0.7, below the 1-80 range of the Gleeshan and McMahon test data, their drag formulas give reasonable results. The estimated power (work per unit time going into the pads, on average) was about 500 W , well below the peak power the subject might be capable of. The observed penetration was about 20 cm , in complete agreement with theory, up to measurement error. Assuming the subject was capable of 1000 W for a short burst, we could have, according to our simulations, reduced the plate area by a factor of two, down to a size that we had deemed too small. Perhaps with practice this would have worked. The buoyancy of the cavity, even after the Gleeshan and McMahon reduction factor, accounted for an average lift of about 500 N . The buoyant term was so large that the plate is predicted to come almost to a stop in the constant foot-force simulation.

How is this concept different from, say, hopping from floating boat to floating boat? In this case the buoyancy of the plates is a purely dynamic effect. You cannot stand on them (see online videos). The support comes from the inertia of the water under them, from the hydrostatic effect of the transiently opened cavity and from fluid drag, in analogy, as mentioned, with a snowmobile on water. It uses exactly the hydrodynamic ideas of Gleeshan and McMahon for the lizards, taking literally their idea of neglecting the forces to lift the feet. People in Northern sea areas have commented on jumping from ice 'flake' (a small flat ice berg) to ice 'flake' in late winter when the ice breaks up, jumping on flakes which they claim would sink if they stood on them. This is really the same effect as we see here, with perhaps a bigger contribution from the inertial term. There are also Shaolin (Konfu) munks who have developed a skill of running on a similar set of boards; they claim it takes many years to master [4].

Our subject reported a sensation similar to that of running up hill. In fact, if the plate velocities can be modeled as moving at constant speed during the contact phase, then the situation is, with a Galilean change of reference frame, identical to running up an escalator, slapping the feet on each step.

Have we really demonstrated, and explained, running on water? It's a matter of definition.

## Materials and Methods

Experiments. The fit 39 y.o. male subject's weight was $F=830 N$. Because the empirical drag constants have a few percent error and we are applying them to a square rather than a circle, we add negligible error by using $g=10 \mathrm{~N} / \mathrm{kg}$ and $\rho=1000 \mathrm{~kg} / \mathrm{m}^{2}$ in all theory and experiments. The pads were made from 9 mm thick plywood with $A=1 \mathrm{~m}^{s}$. The keels were boards $(129 \mathrm{~cm} \times 21 \mathrm{~cm} \times 2.4 \mathrm{~cm})$. The ' X ' shape of the keel was so that for vertical motion the ideal drag calculations should be little altered (Fig. 1a. The total wood volume of $0.022 \mathrm{~m}^{3}$ displaced 220 N had a total dry mass of 10.5 kg . Because the wood was unpainted and absorbed water its mass was an unknown amount higher than 10.5 kg and the bouyant force something less than $220 \mathrm{~N}-105 \mathrm{~N}=115$ N . This is a small but non-negligible contribution to the needed support of 830 N . The subject's chosen running period was $t_{s}=0.40 \pm .04 \mathrm{~s}$ per step, longer than the 0.25 s of sprinters. The pads sank about $h_{\max }=20 \mathrm{~cm}$ during each $t_{c}=.3 s$ contact time.

Theory. To estimate forces and power required we use the formulation of Glasheen and McMahon [2,5,6] reviewed in Minetti et al [1]. A foot pressing into the water is modeled as a round horizontal plate slapping and then penetrating. We write the net force $F$ as a sum of three terms,

$$
\begin{equation*}
F=\overbrace{\underbrace{\text { Hydrostatics }}_{C_{H}}}^{0.7 \cdot \rho g A \cdot h}+\overbrace{\underbrace{0.35 \cdot \rho A}_{C_{D}} \cdot \dot{h}|\dot{h}|}^{\text {quadratic Drag }}+\overbrace{\underbrace{\frac{4 \rho A^{3 / 2}}{3 \pi^{2}}}_{C_{M} \approx 0.135 \rho A^{3 / 2}}}^{\text {added Mass effect }} \cdot \ddot{h} \tag{1}
\end{equation*}
$$

with $h, \dot{h}$ and $\ddot{h}=$ depth, downwards velocity and acceleration of the plate from its start at the surface, respectively; $\rho=$ water density; $A=$ plate area; and $C_{H}, C_{D}$ and $C_{M}=$ coefficients for hydrostatic, $\underline{\text { drag }}$ and added mass terms, respectively. The quadratic drag proportional to $\dot{h}^{2}$ is written as $\dot{h}|\dot{h}|$ so as to have the right sign in case of direction reversals. Even though the plate has a neglected volume, in the time period before water rushes on top of it, which is assumed to be the full duration of the stroke in this model, the hydrostatic force is the weight of the displaced fluid. However, the wave dynamics make cause a non-constant coefficient of drag which Glasheen and McMahon found they could well-fit assuming a height-independent drag and a reduction in the hydrostatic term (from $\rho g A h$ to $0.7 \rho g A h)$. If the plate were circular, the added mass $\left(0.135 \rho A^{3 / 2}\right)$ is about two thirds of the volume of the half sphere under the plate. The three coefficients $(0.7,0.35$,
and $4 / 3 \pi^{2}$ ), although partially checked against theory [2], may be regarded as empirical constants. We use two models to estimate the velocity, penetration and power required.

Constant $\dot{h}=v$ model. Assume the downwards stroke is at constant velocity during contact, then there is a slapping impulse $c_{M} v$ with an average force during contact of $c_{M} v / t_{c}$, a constant drag $c_{D} v^{2}$ and an average hydrostatic lift of $c_{H} v t_{s} / 2$. These must add to give the average contact force $F=F_{s} * t_{s} / t_{c}=1110 N$ where $F_{s}=830 N$ is the weight of the subject. We can solve the resulting quadratic function to find $v=0.64 \mathrm{~m} / \mathrm{s}$. We can also find the average power as the sum $P_{\text {ave }}=c_{M} v^{2} /\left(2 t_{s}\right)+c_{D} v^{3}\left(t_{c} / t_{s}\right)+c_{H} v^{2} t_{c}^{2} / 2 t_{s}=$ 460 W . The penetration distance is $h_{\max }=v * t_{s}=0.19 \mathrm{~m}$. The support force is born $60 \%$ by hydrostatic terms (the weight of the open cavity), $25 \%$ by the collision with the water added mass, and $15 \%$ by the quadratic drag term. The Froude number is about $v^{2} / g r=0.7$, just below the very bottom of the range tested by Gleeshan and McMahon (Froude numbers from 1 to 80 ).

Constant $F$ model. Assuming the foot contact force is constant during the contact time $t_{s}$ we can find $h(t)$ by numerically solving the differential equation: $F=C_{H} h+$ $C_{D} \dot{h}^{2}+c_{M} \ddot{h}$ with the initial conditions $h(0)=0$ and $\dot{h}(0)=0$ in the time span $0 \leq t \leq t_{c}$. With that solution we then find the penetration $h_{\max }=h\left(t_{s}\right)=.21 \mathrm{~m}$, the peak velocity $\max (\dot{h}(t))=.92 \mathrm{~m} / \mathrm{s}$ and the average power $\int_{0}^{t_{s}} F \dot{h} d t / t_{s}=F \cdot h\left(t_{c}\right)=570 \mathrm{~W}$ (code available online).

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## References

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## Figure Legends



Figure 1. Experiment: running on pads. (a) The bottom of a pad, showing the keels which resist side slip, (b) The 10 pads set out in the pool, the red dots are to aid the subject in foot placement, string holds them in line, (c) a pad and foot at the end of a step. The open gap is seen as is the water, beginning to rush in.


[^0]:    ${ }^{1}$ Don't try this at home.

