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FORCES IN BICYCLE PEDALLING

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ABSTRACT

Although pedalling is virtually the sole successful method of powering bicycles and other human-powered vehicles, and pedal-ergometry is the foundation of much work-physiology, it appears that details of the activity are not well understood. Better comprehension could be important in addressing: the optimal load characteristics and pedalling style for increasing endurance or power; the design of appropriate pedaller restraints, and of efficient structures for pedalled devices; and the consistency and significance of exercise tests. The purpose of this paper is to discuss some mechanics-related issues, typically arising from a simple kinematically determined model of the leg, which appear not to be generally acknowledged.

INTRODUCTION

Pedalling seems to be a relatively simple activity, as contrasted with running for example. However, as pointed out by Gregor et al. (1), and somewhat implied by Hull and Jorge (2), and Bolourchi and Hull (3), the biomechanics of pedalling is not yet fully understood. There is no consensus about what characteristics are desirable in a pedalled device, or what loads may be expected, or how the pedaller should control muscle tensions to produce power with the greatest efficiency and comfort.

A number of studies such as the above-mentioned (and others cited by them) have made progress in describing pedalling mechanics. Pedal forces, sometimes seat and handlebar forces, and even electro-myographic evidence of muscle activity have been measured. Based on these results, on mechanical theories of muscles, and on hypotheses about optimal pedalling, some investigators have attempted to find the best pedalling style (Redfield and Hull, (4)), and even to define improved pedalling motions (Miller and Ross (5), Okajima (6)). These efforts are a good beginning, but they do not furnish explanations for most observations, and leave a number of issues unaddressed.

In this paper we will treat a few somewhat disjoint topics, which may help in understanding pedalling, and which will perhaps stimulate a more critical examination of popular beliefs. The idea is *not* to present detailed quantitative analyses of reality, but rather to make some observations about simple special cases, which it is hoped may remain qualitatively correct even when the idealized models don't apply exactly.

The first section deals with the effects of gravity forces and inertia forces acting on the leg. For example, one might assume that the leg's muscles are responsible for accelerating and lifting the leg, and might therefore try to minimize the muscular effort required. However for a simple model leg with a locked ankle, acting on a velocity-controlled pedal (or equivalently, a pedal connected to a large inertia), it turns out that the leg's weight and inertia do not affect the muscle tensions. The work of creating the leg's kinetic or potential energy is supplied by the pedal, and then is reabsorbed by the pedal at another part of the cycle. The standard ratcheting freewheel has little effect on the situation, if two legs contribute.

The second section briefly addresses the notion of muscular 'efficiency', and points out that it might most easily be studied through the kinematically determined leg of the first section. Some possible flaws in using 'Hill'-type muscle descriptions for this purpose are mentioned. The balance of the section is predominantly a criticism of the following idea: that the force exerted by the foot should act in the direction of pedal motion for greatest efficiency, or that any component of foot force perpendicular to the pedal's path is 'wasted'. Firstly, it is pointed out that even if an 'ideal' foot-force direction were known, superposed gravitational and inertial forces would generally modify it. Secondly, for a kinematically determined leg with simple muscles, in the absence of gravity or inertia, it can be shown that pedalling with the foot force tangent to the motion would frequently lead to the performance by one muscle of negative work, which is highly inefficient.

Section three: some loads which act on the bicycle during pedalling have been measured or deduced by other workers, but have not been divided into meaningful components or explained conceptually. Some generic components of pedaller-loading are described here, and explained as far as possible. Some support reactions provided by the seat, handlebars, and pedals are outlined.

1. GRAVITY AND INERTIA

Importance of Muscle Tensions

In the study of pedalling, two kinds of forces are of particular interest: the joint moments (or more fundamentally, muscle tensions); and the forces exerted on the pedals, seat, etc. It seems likely that questions of efficiency, power production, maximum exertable foot force, and so on can be answered only with reference to the muscle tensions, however these are very difficult to measure directly. The more accessible forces on the pedal, seat, etc. can be used to *help infer* muscular tensions, but in their own right they appear important only for structural design, and for deciding what support is required by the pedaller. So it seems that a significant question is how to deduce the muscle tensions, or at least to know what could affect them besides the pedaller's volition.

Perhaps the simplest pedalling problem one might study is that of a weightless leg, fixed at the hip, and pedalling slowly — that is, neglecting gravity forces and inertia forces. The question posed here is, how would gravity and inertia complicate the analysis? The perhaps surprising answer (for a conventional bicycle, with its large apparent inertia at the pedal, and limited ankle flexion) is that joint moments — equivalently muscle tensions — may be almost unaffected by these 'external' forces. Gravity and inertia loads are essentially carried by the seat and pedal, in addition to any muscle-derived pedalling forces already present. If this can be confirmed in practice, it might be reasonable to focus on the inertia-less gravity-free problem when deciding how to pedal efficiently; and when studying measured pedal forces, one might measure and then subtract the components attributable to gravity and inertia, and know that what remains is due only to the muscles.

The argument is based on the leg approximating a *kinematically determined mechanism* — one in which the position of the pedal, and the positions (and thus the angles) of each joint, are given as functions of time, independently of any muscular, gravitational, or inertial forces. Obviously this is never strictly true for any real human or machine, but we will discuss below how appropriate it may be as an approximation to a leg.

'External' Forces on a Kinematically Determined Leg

Consider a pedalling leg depicted as a linkage of jointed bars in Fig.1. To render this kinematically determined, the links (thigh, lower leg, foot, and pedal-crank) must be perfectly rigid, the hip-joint must be fixed relative to the pedal-crank axis, the pedal-crank's rotation must be prescribed as a function of time, and one further constraint is required to remove the ankle's freedom to flex arbitrarily. Some possibilities include immobilizing the ankle with a 'ski boot', controlling the foot's orientation with a 'guide rod' (shown dashed in Fig.1), or (if no major forces act on the foot itself) connecting the pedal directly to the lower leg, or placing the pedal axle at the ankle joint. For definiteness, only the first of these will be considered in what follows.

The relevant feature of a kinematically determined leg is that any *external* forces acting on the rigid links can be carried by joint *forces* with no need for joint *moments*. In particular, we have here a thigh segment and a lower-leg segment, pinned to each other at the knee, and pinned to support points at the hip and foot (Fig.2). Adding any 'external' forces leaves any pre-existing joint moments (or muscle tensions) unaltered. 'External' forces here include gravity forces, and inertial (D'Alembertian) forces which are equivalent to to external forces. On the other hand, the force on the pedal *is* changed, even in the direction of motion. The pedal is therefore responsible for supplying energy to lift the leg; but since it also re-absorbs this energy when the leg falls, there is no effect on the average power produced by the leg.

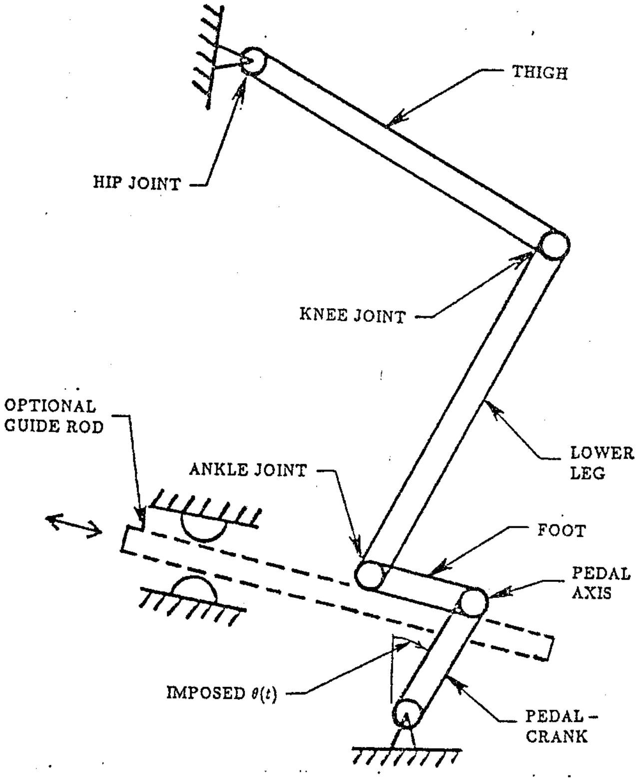

FIGURE 1 The linkage representing the leg plus the position-controlled pedal-crank is not kinematically determined. It can be made so with either a guide rod fixed to the sole of the foot (shown dashed), or an ankle-immobilising "ski-boot" (not shown).

The point here is simply that for a kinematically determined leg, 'external' forces have no direct mechanical effect on muscle tensions. It seems then, that in designing the 'best' pedalling strategy that gravity and inertial forces should not be taken into account. If any 'external' force is then added a pedaller may of course *choose*, say, to keep the total pedal-force unchanged — the 'external' force would then be carried by additional moments at the knee and hip. However, such a strategy would probably reduce the efficiency of muscle usage.

The requirement of kinematical determinacy is absolutely essential to the argument. Imagine, in contrast, that the pedal were free to slide radially on the fixed-rotation-rate pedal-crank; or alternatively that the fixed-radius pedal-crank's rotation were governed purely by a viscous resistance (and for example the rider's selection of a constant pedalling velocity). To prevent added external forces from disturbing the foot's velocity, certain components of the foot-force would have to be left absolutely unchanged — and therefore changes in hip and knee moments *would* be required.

Relevance to Actual Pedalling

Now that it has been explained how external forces on an ideal kinematically determined leg are supported without affecting joint moments, the important question is how closely this model resembles an actual pedalling leg.

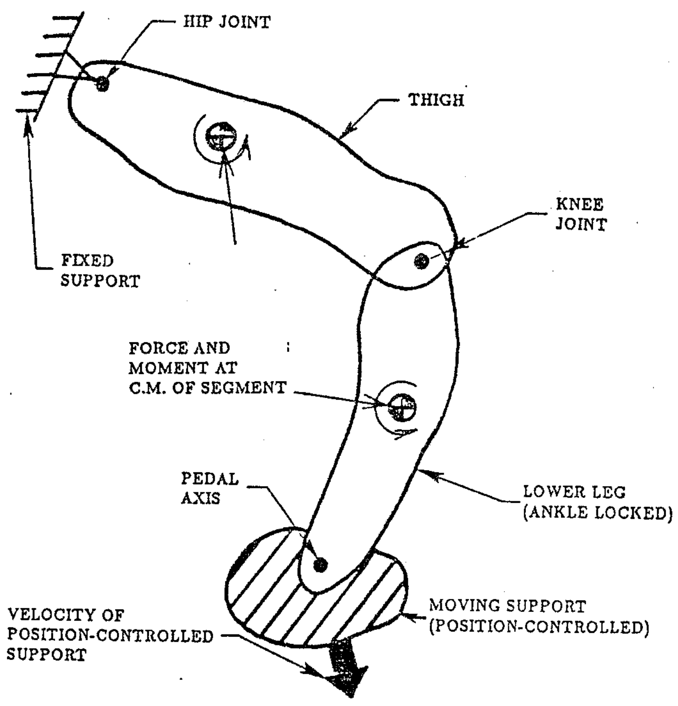

FIGURE 2 Any 'external' (e.g. gravitational or inertial) forces on the segments of this kinematically determined leg are carried by joint *forces* alone. No joint moments are necessary.

(a.) Constraints on foot motion. Near-rigidity has no meaning *absolutely,* but only in relation to typical forces and typical displacements. In the radial direction, the pedal is well-supported (by a stiffness of order 1 kN/cm (500 lbf/in) on a typical bicycle), so that displacements due to pedalling forces of a seated rider are just a few percent of the distance over which foot-force builds up. Resulting variations in the knee's position, angle of flexion, and so on, similarly appear to be negligible. In the tangential direction the pedal's velocity is certainly not fixed, however it is connected (by the same stiff structure which withstands the radial loads) to a large inertia, typically equivalent to 1,000 – 8,000 kg (2,000 – 18,000 lbm) at the pedal, depending on gear ratio. Thus, the fractional change in speed within one cycle in normal level cycling, due to typical 'external' forces or even muscular forces, would also be very small.

It may be remembered that bicycles usually employ a ratcheting over-running clutch (for reasons of safety, convenience, and transmission design), whereas this analysis assumes that the inertia of the bicycle not only absorbs the work of lowering and decelerating the leg, but subsequently lifts and accelerates it. However practically speaking, the analysis would still hold (and the rider would feel free to exert any desired muscle tension) as long as any retarding torque due to the 'external' forces does not overcome the propelling torque of the muscular forces. With two legs turning pedal-cranks rigidly connected 180 degrees out of phase, this is likely to hold true, as one or the other leg is typically in its 'strong' phase, and also the gravitationally induced torque is reduced.[1] However, a single leg has an extensive 'weak' phase in which it is difficult to supply the required energy. To retain the conditions of the analysis if only one leg is pedalling, one might have to store energy during the 'strong' phase which will be returned in the 'weak' phase (by compressing a spring which will lift the leg later, or decelerating the leg in the 'weak' phase), so net power is still delivered through the clutch at all times.

(b.) Kinematical determinacy of the leg. The axial and bending deformations of the thigh, lower leg, and foot are small, so these might reasonably be considered rigid. Also, while stand-up pedalling clearly is not kinematically determined, a hard bicycle seat supports a seated pedaller relatively firmly. However, the ankle joint *is* free to flex, so the real leg clearly *is not* fully constrained. It is suggested here, though, that the human leg in ordinary pedalling can be still considered nearly kinematically determined. The ankle has a restricted range of extension, and the pedal is located quite close to it, so ankle flexion causes little change in the angles of other joints. While it is true that an actual pedalling leg is not quite kinematically determined, external forces may substantially modify only the ankle's moment.

Perhaps the best way to check the applicability of this discussion is by laboratory testing: using a pedal-dynamometer to measure foot-force, and driving the pedals at near-constant speed with a large flywheel or motor. Certainly if the pedaller wears 'ski-boots', and tries to maintain a fixed average-power level, or even exerts no muscle moments, the predictable effects of any added external force or mass should be easy to verify. (Such simple kinematically determined experiments could contribute much to an understanding of pedalling.) Note: since the pedaller's pelvis is actually held against the seat only by body weight, it may have to be further restrained in order to act truly fixed. This suggests an interesting experiment: what level of seat-springiness significantly affects pedal forces?

Without the 'ski-boots', the same test can be tried. If the hypothesized independence of gravity and inertia forces is not confirmed, it might imply a larger role for the ankle than now envisaged by this author.

2. MUSCULAR EFFICIENCY AND FOOT-FORCE DIRECTION

Considerations in Studying Optimal Pedalling

Simply speaking, for a given level of stimulation, individual muscles produce maximum power when their contraction rates are neither too fast nor too slow. (McMahon (7), p. 15) Also, the use of muscles incurs some *metabolic cost* or 'fatigue' (depletion of fuel, buildup of waste products, etc.); presumably this depends on the tension, the contraction rate, their past variations, and so on. Such considerations lead to the question "What is an efficient (or sustainable) pedalling strategy?" (See Forester (8) for a thoughtful discussion bearing on this subject.) Issues include what force to exert at each instant in a pedalling cycle, how fast to pedal, what pedal-crank length and position (relative to seat) is best, whether a non-traditional foot motion could

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[1] The top and bottom of the pedal stroke are relatively 'weak' phases but it is not known whether this invalidates the analysis. The question is whether the legs' mechanical energy increases enough in any part of this region to 'use up' all the muscle torque typically applied. Since it appears that a maximum of potential energy, and (for constant foot speed) a minimum of kinetic energy, are attained in or near these 'weak' phases, the answer depends on the pedalling speed and the sharpness of the total energy peak— or valley.

[Max. p.e. of one leg is not simultaneous with min. p.e. of other] increase power,[2] etc. It is clear from experience that these issues *are* very important, because a seat that is too low, or pedal-cranks that are too short, drastically reduce power output. Fatigue is by no means a function solely of net mechanical work performed.

In studying pedalling efficiency a kinematically determined leg model is particularly useful, for three reasons. *Firstly,* the muscle tensions are unaffected by 'external' forces as discussed earlier. *Secondly,* it is reasonable to think that kinematically determined pedalling has the potential to be very efficient, since muscle tensions are then *independent* and *absolutely arbitrary.* Joint moments need not be constrained to fix any component of the foot force. The pedaller does not have to drive the pedals to prevent stalling during weak parts of the pedal-cycle, and may even rest for several pedal-strokes. (It may be deficiencies in this regard, arising from lack of inertia, which make some exercise bikes feel unpleasant.) *Thirdly,* for a given pedal-cycle, the muscles of a kinematically determined leg have predetermined extension rates at each instant. (This was pointed out by Harrison (9).) The question of efficiency therefore reduces to the selection of muscle tension-levels at each point of the cycle. If appropriate 'cost' functions were known, it would be relatively straightforward to select optimal muscle tensions throughout the cycle, so as to produce a given power at least 'cost'. It should even be reasonably easy to find speed-variations which increase 'efficiency'. Alternately, one might seek to achieve absolute maximum short-term power. Once the problem is defined, there is scope for investigations of a fairly general nature. In view of the advantages of the kinematically determined model, it is hoped that the ankle of a real leg can be treated as a perturbation of a kinematically determined leg.

It is important to mention some concerns about muscle-force laws, and how they are used. A version of Hill's equation (McMahon (7), pp. 13-15), which relates muscle tension to contraction rate, is often adopted. However this relation is for a constant level of stimulation ("maximum voluntary contraction", according to Harrison (9)), not for various fixed 'rates of metabolic cost'; so while it might serve in a search for maximum short-term power, it may not help in achieving minimum tiredness in a long-term effort. Further, it is not intended to form a complete mechanical description of muscle behavior — the equation's 'tension'-parameter depends on the muscle's degree of stretch, and additional nonlinear 'parallel' and 'series' springs should be included (McMahon (7), pp. 6-16; Astrand and Rodahl (14), pp. 101-111). See Harrison(9) for calculations involving a series compliance, and see Zehalak (10) for a more sophisticated mechanical description of muscles. Finally, it may be mentioned that two or more 'Hill's equation' actuators, acting across the same joint, need not generally act like a single 'Hill's equation' actuator. In fact, for a single joint angle, power as a function of joint-closing rate could even show distinct peaks (maxima) if the actuators have very different shortening rates. It is similarly an error to assume that the foot of a multi-joint leg with 'Hill's equation' actuators would itself have 'Hill's equation' behavior for each direction of motion.

Significance of Foot-Force Direction

Even without knowing muscle cost-functions, for a kinematically determined leg it is possible to discuss the common idea that foot-force should be directed *parallel* to pedal-velocity for maximum efficiency. This prescription may have arisen from ideas on the best use of a fixed-magnitude force, or on minimizing joint or bearing friction-losses, or perhaps from a notion that any force perpendicular to the pedal-path was due to a separate tensed (but non-working) muscle, which would tire.

The idea can be criticized in two respects: (I) It refers to *total* foot force rather than the part *due to muscle tensions alone,* and so is affected by any external (and inertial) forces. (II) There is a simple example in which enforced parallelism of force and velocity is demonstrably inefficient for any muscle model. So the issue of maximum efficiency in the case of an actual human leg in no sense reduces to a simple requirement that pedal force should be in the direction of pedal motion.

(I): As explained in Section 1 for a kinematically determined leg, if it were actually true that optimal muscle tensions produced "foot-force parallel to pedal-velocity" in some particular circumstances, a heavier leg or tilt relative to gravity would change foot force without affecting muscle tensions. "Foot-force direction" therefore has no unique meaning in terms of muscle tensions. A statement with only one interpretation might be that the muscular force (foot-force arising from muscle tensions only) *should be parallel to pedal-velocity.* (Along the same lines, it might be helpful to the study of pedalling mechanics if published measurements of pedalling force routinely included plots of the muscular force.)

(II): Even if the prescription is restated in terms of muscular force, however, it can be shown that it often causes one muscle of a simple (kinematically determined) leg-model to perform *negative work.*[3] In this case greater efficiency is always achieved by switching this muscle off (or perhaps even activating its antagonist), and permitting the force to have some component perpendicular to the pedal-velocity.

(a.) Negative work for some foot motions. Fig.3a shows a leg modeled as a set of rigid bars, pinned together, with idealized muscles intended to be functionally equivalent to those of the leg. (The picture could be improved if we had the information to represent some measure of muscle strength as a thickness, and to give each muscle proportionally scaled moment-arms for each joint it crosses.)

The leg model we will actually explore (Fig.3b) is made kinematically determined with a constant pedal-speed, and with a ski-boot ankle brace to prevent ankle flexion. (Note that this is not the same as simply placing the pedal-axle at the ankle: if the gastrocnemius is used to apply a flexing moment about the knee-joint, then the tibialis anterior and associated muscles would have to resist its moment about the ankle joint.) This model leg will be considered to have two independent 'actuator-muscles' — a hip actuator (composed of the gluteus maximus and the iliopsoas) and a knee actuator (composed of the biceps femoris (short head) and gastrocnemius, and the vasti). These

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[2] There is no obvious reason, apart from mechanical simplicity, why a round, constant-speed pedal path should be preferred (Harrison (9)).

[3] Negative work occurs when a muscle exerts tension while lengthening non-elastically; an external agent is performing unrecoverable work on the muscle. This is undesirable for two reasons: the work (which presumably was muscular in origin) is simply lost; and the absorption of energy incurs some metabolic cost. (See McMahon (7), p. 34, pp. 211-214.) For a kinematically determined leg, negative work is *always* inefficient — if that muscle were unstimulated, the net work produced by the leg would be greater, and the metabolic cost would be lower.

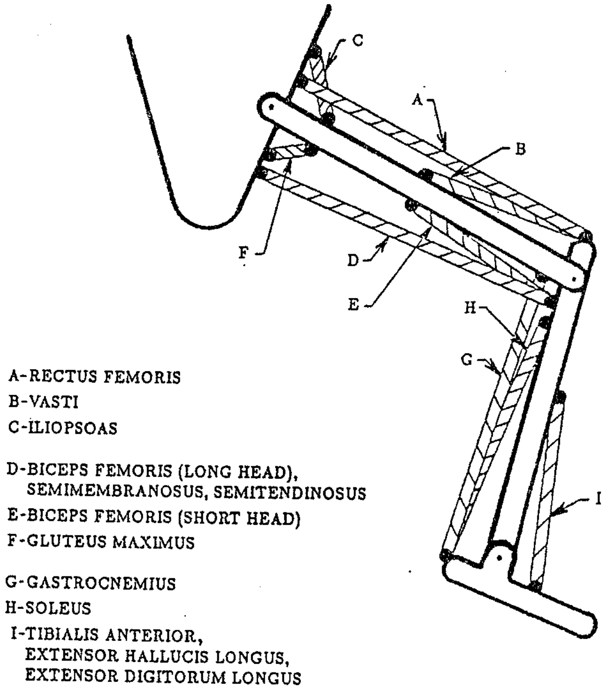


FIGURE 3a A stylized functional representation of the major muscles crossing the hip, knee, and ankle. Additional terminology: A+B = QUADRICEPS, D+E = HAMSTRINGS, and G+H = TRICEPS SURAE.

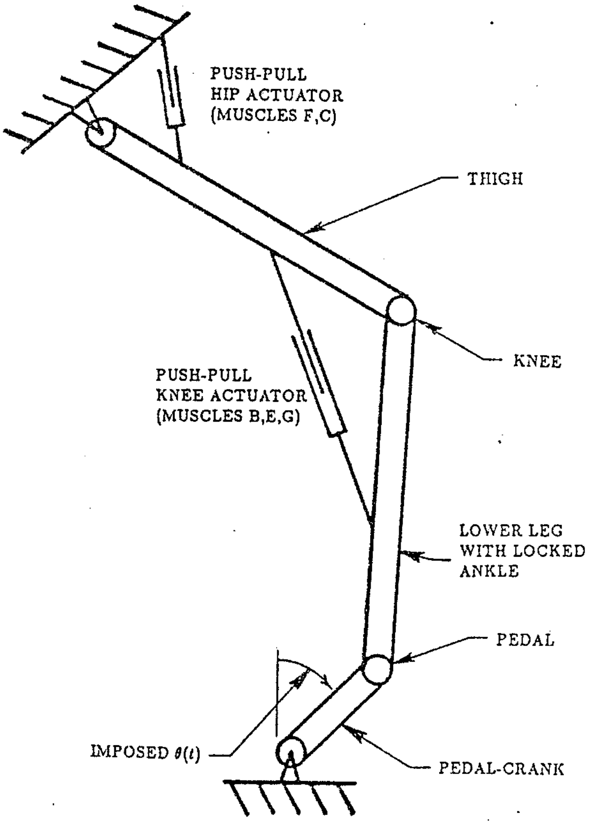


FIGURE 3b With the ankle locked, the lower leg and foot are kinematically equivalent to a bar from knee to pedal. Push-pull single-joint hip and knee actuators model the function of the muscles indicated in parentheses.

actuators can provide moments of either sign, to flex or extend (close or open) their respective joints. Gravity and inertia are ignored per Section 1, and to simplify the discussion the elasticity of the muscles has also been neglected.

In this leg, pedal positions and leg configuration are controlled at all times. In contrast, foot-force (which arises solely from actuator moments) is controlled by the pedaller: as long as the pedalling period is long compared to the time for muscle-force buildup, force can be applied in any direction, with any magnitude permitted by the maximum actuator moments. Thus it may be made parallel to pedal-velocity, or purely vertical, or perpendicular to pedal-velocity, etc.

At some instant in a pedalling motion, consider the foot velocity (direction and magnitude) and foot-force exerted (direction and magnitude). From the *velocity*, we can uniquely find the extension (opening) rates of the two joints — in fact, their signs are defined by the *velocity direction* alone. Likewise, from the *force direction*, we can determine the signs of the joint moments applied by the two actuators, which are considered positive if they tend to extend the joints. The general claim is: for many foot velocity directions (or pedal-path orientations), to demand that the force be exerted parallel to the velocity means that *one actuator will be ineffective — it will absorb work.* That is, one joint's extension rate and extending moment will be opposite in sign, so it performs *negative work.*

As an example see Fig.4: if the foot were moving in the direction of A, joints H and K would be extending (opening). If the force exerted by the foot were controlled to lie along A, there would have to be actuator moments trying to extend both joints — this can be seen because the reaction force of the pedal on the foot, which acts in the -A direction, exerts a flexing moment about both joints. Both actuators are thus doing positive work — there is no absorption. (Motion A extends H and K, and a force along A implies actuator moments *trying to extend H* and K.) On the other hand, motion along B similarly extends H and K, but directing foot-force along B means that the actuator at H is *trying to flex* (close) the hip — that is, it absorbs work. As mentioned above, this is inefficient.

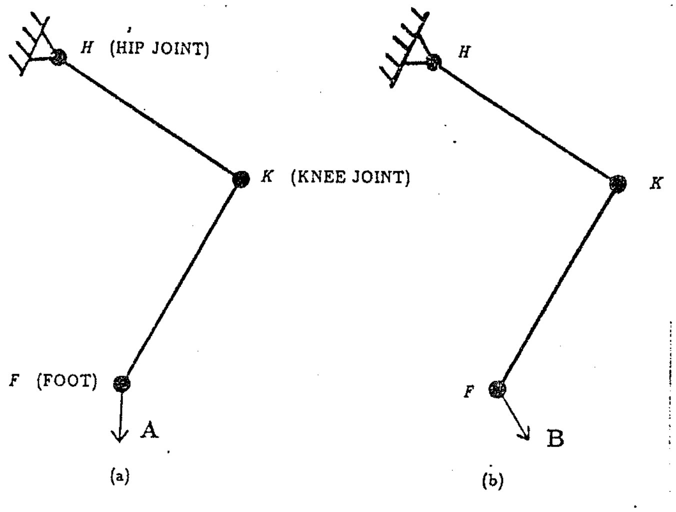


FIGURE 4 The idealized leg of Fig.3b is represented in two circumstances: (a.) The foot moves in the direction of A, exerting force purely in that direction; both hip and knee actuators produce power. (b.) The foot moves in the direction of B, exerting force purely in that direction; the knee actuator produces power, but the hip actuator *absorbs* power (performs negative work).

(b.) All foot motions causing negative work. For this simple leg in any configuration[4], it is possible to map out exactly the velocity directions for which a parallel force would imply that one actuator must absorb energy (which was originally produced by the other).

The first step is to find, as the direction of the foot velocity is varied, when each of the two joints attains zero extension rate. The foot-velocity directions where either joint has zero extension rate are boundaries in foot-velocity space between extension and flexion for that joint. See Fig.5a.

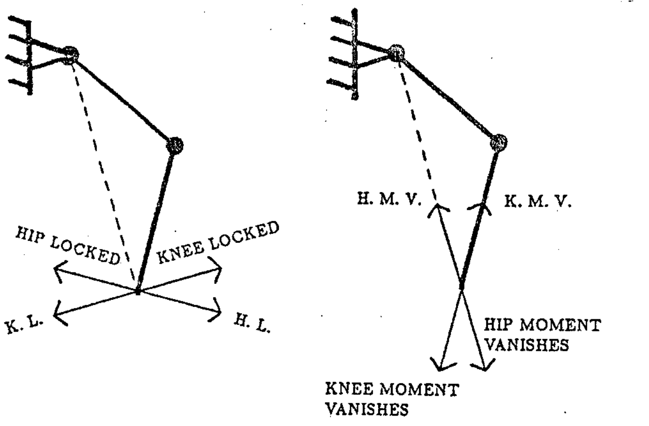
The second step is to find the directions of *foot force*, such that one or other joint moment vanishes. These are boundaries in foot-force space between moments tending to flex and moments tending to extend either joint; Fig.5b.

It is not just coincidental that the lines constructed in Fig.5b are perpendicular to those in Fig.5a. For example, if we lock the knee, and require the hip moment to vanish, then the foot can do no work. This means that the only force it can exert is perpendicular to its only possible motion — the force direction for which the hip moment vanishes (i.e. for which the force is due to the knee alone) is therefore perpendicular to the velocity direction for which the knee-joint has zero extension rate.

The third step is to superpose the two diagrams (Fig.5c). The four lines divide the set of force / velocity directions into eight sectors. If in one of these sectors parallelism of foot-force and foot-velocity means that both joints are producing positive work, then in the two adjacent sectors one of the joint moments or extension rates will have changed sign, leading to negative work in that joint. Further, passing on to the next sector still, the joint performing negative work must revert to positive work. (The sum of the powers from the two joints must be positive, because the net leg power — foot-force times foot-velocity — is positive at all times.) This means that the two boundaries, of a sector where one joint performs negative work, refer to conjugate quantities for the *same* joint, and therefore that the two boundaries for a sector without negative work must refer to *different* joints (since no three consecutive boundary lines can refer to a single joint). It is possible to see, then, that the acute angles in Fig.5a and Fig.5b (which are never divided when the two diagrams are superposed) must represent 'good' sectors, as their boundaries refer to similar quantities for different joints. These good sectors are separated by 'bad' sectors, in which one joint or the other absorbs work.

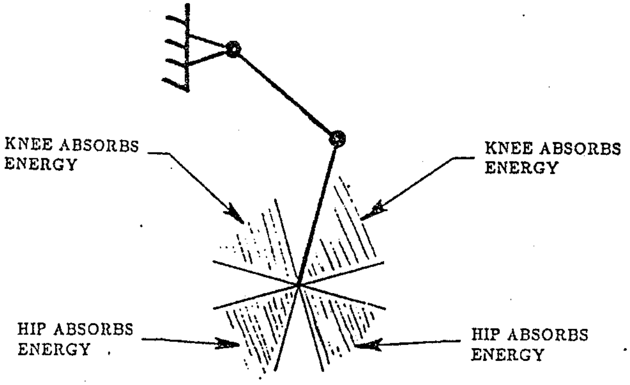
In the case where no angles in Figs.5a,b are acute (i.e. all are right angles), the 'bad' sectors vanish. This is the case only for a leg whose hip-to-foot line is currently perpendicular to its knee-to-foot line. The thigh is then the hypotenuse of a right triangle, which is possible only if it is longer than the lower leg.

This development is closely allied to another important result, for a two-joint, two-actuator leg with foot constrained to move along a fixed path: If the actuators each exert their maximum force so as to produce maximum power individually, the component of foot-force along the foot's path is maximum, but the *total* foot-force is generally *not* parallel to its motion. On the other hand, if the actuators co-operate to exert the maximum foot force, entirely *parallel* to the foot's path, of which they are capable, one will generally be at less than its maximum force (or power), and as shown above may even be absorbing power. Thus, the maximum force-component parallel to the given foot path is almost always *decreased* if perpendicular components are prevented. This conclusion is foreshadowed by the work of West and Asada (11), on the advantages of adding kinematical constraints to (or equivalently removing statical constraints from) a robot manipulator.



(5a) MOTION DIRECTION WHEN EITHER JOINT IS LOCKED

(5b) FORCE DIRECTION WHEN EITHER JOINT EXERTS NO MOMENT



(5c) IF FOOT VELOCITY IS WITHIN A SHADED SECTOR, AND FOOT-FORCE IS PARALLEL TO MOTION, NEGATIVE WORK IS PERFORMED BY THE INDICATED JOINT

FIGURE 5 Conditions in which negative work is performed by either joint of a simple leg.

(c.) Negative work in a complete pedalling cycle. In the foregoing, the possibilities for negative work with the leg in a given configuration were treated. However, more important are the prospects for negative work over the whole pedalling cycle (if force is held parallel to velocity).

In a typical smooth, convex foot path (such as the standard circle), this simple leg would pass through eight regions, in four of which *tangential foot force* implies *work absorption at one joint or other.* One way to see this would be to construct the diagram of Fig.5c at each point of the pedalling circle, and then decide whether the foot velocity is in a 'bad' direction or not. But unless the pedalling circle is very small, it is easier simply to note the points on the circle at which either joint's extension-rate or moment changes sign, and realize that these delimit good and bad regions much as in Figs.5a,b.

Kinematically, the hip joint reaches its furthest excursion at points 2,2' when the crank is aligned with the knee-foot line; see

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[4] That is, as long as any direction of foot-motion is possible — the lower leg must not be parallel to the thigh.

Fig.6. The knee joint reaches its limits at the two points 1,1' where the crank is aligned with the hip-foot line. These points are analogous to the boundaries in Fig.5a, in that one joint is not extending. Assuming the foot force is directed tangent to the foot path, the hip moment vanishes at the points 3,3' when the foot reaction force (assumed tangent to the circle) passes through the hip. The knee moment changes sign at the points 4,4' where the lower leg is tangent to the pedal-circle. These points are analogous to the boundaries in Fig.5b. For this simple cycle, the regions with work absorption are again bounded by two transitions relating to the same joint. (For a more wiggly path, the *same joint* could move in and out of absorption, so this criterion for determining 'bad' regions is not general.)

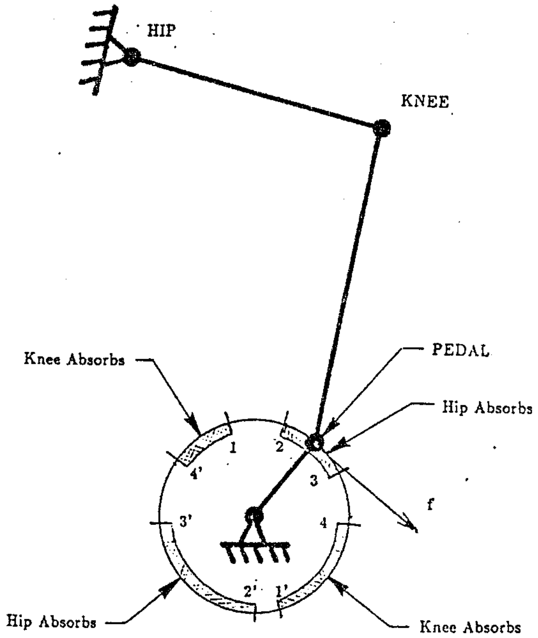

FIGURE 6 Representation of a simple leg with human dimensions, its ankle constrained by a "ski-boot", pedalling with the use of knee and hip actuators only. (The resisting load is not shown.) It is assumed that the foot's force f , in the absence of gravity and inertia, is maintained parallel to its instantaneous velocity. At the numbered points, joint moments or extension rates change sign. Consequently, in each shaded sector the indicated joint *absorbs* energy (performs negative work).

The conclusion is, that for a particular simple leg-model, if the foot-force is required to remain nonzero and precisely parallel to pedal-velocity, then inefficiency will inevitably follow. Another way of stating the same conclusion is that if the joints are permitted to perform positive work only (i.e., each joint's moment is required to be exerted in the same sense as its extension rate), then it is frequently impossible to exert foot-force parallel to pedal velocity.

(d.) Two-joint muscles. Two-joint muscles add some conceptual complications, but in many respects the same conclusions hold. For example, if one or both of the push-pull actuators of Fig.3b cross two joints, results analogous to Fig.5 follow exactly (with "actuator" substituted for "joint").

However, a pair of two-joint muscles cannot generally be treated as a single push-pull actuator. The rectus femoris and biceps femoris (long head) do not 'oppose' each other as single-joint antagonists do — they are almost certainly *kinematically independent.* (That is, fixing the length of one does not prevent shortening of the other, so there is a range of foot motions in which both contract, probably corresponding to straightening of the leg.) Instead they must be treated as distinct tension-only actuators, which can exert force only within a sector of directions. If the foot is held fixed, the sector boundaries are the directions of the force of the foot on its support when either muscle alone exerts tension. If this angle is obtuse, as would be the case for the leg of Fig.3a, two 'bad' regions should exist within it, as in the obtuse angle of Fig.5a.

If two-joint muscles are involved, it is no longer possible to detect negative muscular work by looking at the work of individual joints. For example if the biceps femoris (long head) exerts tension while the knee is extending, the negative work of the knee would appear to mean that the muscle was absorbing energy. With the hip locked this would be true; but if the hip too is extending sufficiently, the muscle itself is shortening and performing positive work (equal to the sum of the positive and negative work for the two joints). The apparent negative work of the knee in this case does not imply negative work for any muscle. To avoid negative work in a given motion, only the muscles which shorten should be tensed.

In the context of this brief discussion, the observations by Gregor et al. (1) that both the quadriceps and the hamstrings contract simultaneously in part of the pedalling cycle, and that the knee sometimes exerts a flexing moment while extending, may not be so surprising, depending of course on which muscles are shortening at the time. For future work in this area, it is important to have data on the lengths of two-joint muscles in each leg configuration. (For one muscle this might take the form of a series of foot paths, each representing a given length of the muscle.)

Unfortunately, the simple two-actuator example discussed in this section is only suggestive: for a real leg with its greater number of muscles from which to choose, and its muscle elasticity, it may in fact be *possible* to have force / velocity parallelism without negative work. But the question remains: why should we expect this strategy to be good?

We can reasonably say "don't exert muscular force entirely perpendicular to motion", because this is *guaranteed* to be inefficient.[5] And if the whole leg has only a single active muscle, a reasonable cost function might lead to a strategy of keeping the muscular force within 45 degrees, say, of the direction of motion (i.e., to switch off when mechanical advantage is low), because of low muscle contraction rate, and/or joint friction. But prescriptions more stringent than these require justification.

3. MAJOR FORCES ON BICYCLES DUE TO PEDALLING

Perhaps because the pedaller can deform and also move rela-

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[5] If the foot's force is exerted perpendicular to its path there *must* be negative work (or exceptionally, non-contracting tensed muscles). In view of this, it is initially surprising that Davis and Hull's (12, fig.8a) measurements display a significant amount of vertical foot-force just past the bottom of the pedal-stroke. However it is actually plausible that the muscular force could be zero: the measured level of 80-120 N (18-27 lbf) is reasonably consistent with the weight of a resting leg's foot, which experimentally seems to be relatively independent of pedal position.

tive to the bicycle frame, because of the variety of pedalling styles and rider force options, and because of an imperfect understanding of bicycle dynamics, there seems to be no clear qualitative description of the major force-systems applied to a biycle. As a particular example, the second bicycle loading described in G. G. Klein's U.S. Patent 4,500,103 on stiffer bicycle frames appears impossible to achieve in actual riding. Below are outlined rough descriptions of what I think are some of the major loads associated with pedalling (as contrasted with steering, braking, collision, etc.). They have not been verified experimentally.

Gravity Loads and Pedalling loads

A distinction is made between weight-related loadings and power-related loadings, because each arises in different circumstances, and is affected by different factors. These loadings can further be divided into in-plane, and out-of-plane (which will generally cause higher stresses, and are more easily analyzed separately).

In-plane weight-loads are relatively simple except for the redundancies in rider support by seat, handlebars, and pedals. (How much weight is carried by the seat, and how much by the pedals?) Out-of-plane weight loads are restricted: a rider travelling straight exerts essentially no net moment about the bicycle track. (This isn't true for a stationary exercise bicycle, on which the rider can exert dynamical forces and moments impossible in cycling, by movement of the upper body.) In holding the bicycle frame tilted from the vertical, while of course the overall c.m. remains vertically over the bicycle track, a rider produces a lateral loading composed of lateral forces where the wheels touch the road, and equilibrating loads where the rider touches the bicycle. These loadings are not essentially pedalling-related: even though the rider may rock the bicycle from side to side when pedalling standing-up, this tilting is not essential to the production of power, and conversely tilting can be performed when no pedalling is taking place. (The other major lateral loads acting on the wheels, which arise from steering transients but not from steady turns, are not discussed here.)

Superposed on these weight-loadings, which are henceforth conceptually subtracted out, are loads inherent in power-producing pedalling. These can be very large when accelerating from rest, perhaps ten times their magnitude in steady level riding. In-plane, the main feature is that of restraining the rider from flipping backwards due to the pitching moment about the pedal-crank axle. The more important out-of-plane pedalling loads are apparently less well understood; the essential feature is that the forces producing the crank-turning (pitching) moment are applied to points offset on opposite sides of the bicycle. This means that when the pedal-cranks are roughly horizontal,[6] pedalling forces unavoidably apply a large rolling-type moment to the bicycle frame (and in the opposite sense, to the rider). The question is, what other forces between the bicycle and rider serve to prevent the two from rotating dynamically in opposite directions?

Possible Reactions to In-Plane Moment Vector

Seat reaction in seated pedalling. One major way of balancing this rolling moment is through a lateral force at the seat, which is found in seated pedalling when hand forces are moderate. When standing slightly and grasping the handlebars at their center, the cyclist can feel the seat pressing against the inner thigh of the descending leg. Actually, equilibration of a moment requires not a net force but a couple, so there must be an opposing lateral force applied by the feet to the pedals. In essence, if one leg is exerting all the force, that force must pass through the seat (abductor tension is required). One can simulate this loading experimentally, by rigidly supporting the bicycle-frame in the vicinity of the pedal-crank axis, and applying a side-load to the bicycle seat.

The simplicity of the planar description in earlier sections may be retained if the seat is built like a chair (as it is on many recumbent bicycles). In this case the force of the foot may remain in the plane of the leg, and thus elicit a reaction force at the top of the leg. This means the seat must apply a moment to the frame, which will be in pure bending if the leg-plane is parallel to the frame-plane.

Handlebar reaction in standing pedalling. In standing pedalling with no seat contact, the rider has the option of relying again on a couple of lateral forces (one at the handlebars and one at the pedals); or on a pure moment at the handlebars, which must be predominantly *perpendicular* to the steering axis. One may observe that it is relatively easy to to apply an adequate moment with little side-force (by permitting the hand to contact only the upper and lower surfaces of the handlebar tube), but difficult to provide a sufficiently large side-force. (This can be checked by placing hands together at the center of the handlebar, so the twisting moment is small.) A couple of hand-forces which are roughly parallel to the steering axis is therefore probably dominant, with some moderate side-force in addition. Experimentally, this may be simulated by again rigidly supporting the bicycle frame at the pedal-crank axis and holding it sideways, parallel to the ground. A rod along the steering axis carries a loading weight. If the reference condition is without a weight, then adding it at the 'top' end of the rod provides a force and a moment. If the reference condition is with the weight at one end of the rod, shifting it to the other end is like adding a pure moment. (This is essentially the first loading of Klein.)

Stand-up pedalling is very different mechanically from seated pedalling, because the hip-joint is not fixed in place. Pedalling proceeds by quickly transferring nearly all the rider's weight to the uppermost pedal as it passes the vertical; then straightening the leg as permitted by the drop of the upper pedal and rise of the lower; and completing the descent with the leg locked. (The twisting moment applied to the bicycle thus varies in time like a square wave.) In no sense is stand-up pedalling kinematically determined, so the results of Sections 1 and 2 are not relevant.

When employing any of these reaction strategies, the pedaller could perhaps perform useful work with some of the supporting muscles. This is clearly the case if the arms work on the bicycle's tilt relative to the rider (as when some cyclists pedal up a steep hill or accelerate violently).

The above loadings do not seem to be widely known either by researchers or practitioners, and the author knows of little evidence specifically validating them. However they seem to be consistent with a variety of observations concerning structural de-

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[6] The case when the pedal-cranks are more nearly vertical will not be discussed, as the propelling moment is evidently much smaller. Why should this be so? A simple answer might be that because of the large hip-foot distance, the maximum hip moment would easily be exceeded by any large force not passing sufficiently close to the hip.

flections, structural failures, frame-reinforcing techniques, rider pedalling methods, etc. Published bicycle loading measurements generally include too few load components (for example Soden and Adeyefa (13) do not measure side forces, and simply *assume* moment reaction in high-torque, standup pedalling). However Davis and Hull (12), figs. 6a,6b, find a lateral foot-force during the power stroke in seated pedalling which is about one fifth of the vertical foot-force, roughly consistent with the idea that one leg is doing most of the muscular work, and that its foot-force passes through the seat.

CONCLUSIONS

The reasons for undertaking the preliminary investigation described here are that popular explanations of how pedalling should (or does) work sometimes appear difficult to reconcile with the laws of mechanics; while more scientific investigations as a rule have not produced the kind of qualitative understanding which could guide pedallers, or designers of pedalled devices. In this search for sound explanations, however, there is no intent to denigrate any practically developed advances in pedalling, but rather to develop a better understanding as a basis for future progress.

The main conclusions: In section 1., we advance the hypothesis that it is largely unnecessary (and perhaps somewhat misleading) to think of joint moments in seated pedalling as affected by leg weight or inertia. For a wide range of conditions, it is likely that the joint moments simply reflect the pedaller's effort to produce power effectively.

In Section 2., we point out some dangers of assuming that pedalling 'efficiency' depends on exerting foot-force, or even just its muscular component, tangent to the pedal's path. A better criterion might be that only contracting muscles should exert tension. Our ignorance about the effects of many factors on pedalling efficiency suggests some scope for improvements in pedalled devices, if the right questions are asked and the appropriate muscle 'laws' are adopted.

In Section 3., the two major out-of-plane loadings applied by a pedaller to a bicycle frame are suggested to be (a.) a side force at the seat, reacted at the pedal-crank axis; and (b.) a twisting moment, whose vector is perpendicular to the steering axis, applied to the handlebars and reacted at the pedal-crank axis.

SPECULATIONS AND QUESTIONS

Finally, some observations and comments are offered: Efficient pedalling appears to be a very complex co-ordinated activity, requiring split-second control of which muscle(s) to tense (Harrison (9); cf. Astrand and Rodahl (14) pp. 111,113 on exercise in general), to what degree, in various circumstances. Pedalling forces and support reactions must be selected for the structure available to handle them. For example, a standing pedaller's feet must exert a moment perpendicular to the steering axis, and a seated pedaller must usually exert side forces at the pedals. It would not be surprising if very specific training were needed to do well in the variety of styles used almost automatically by experienced pedallers.

Pedalling is widely used in work-physiology testing, perhaps because it appears to provide a well-defined use of major muscles. (See Astrand and Rodahl (14) p. 337.) However, because of the many different muscles which could be used, the indeterminacy of the reactions permitted on the seat and handlebars, diversity in the *type of resistance* (more or less inertia-like in response to variations in foot force, within a cycle or between cycles), not to mention such factors as seat position, pedalling frequency, and thermal environment (Whitt and Wilson (15)), it appears that this ostensibly standard work-method is not very well controlled. A truly controlled exercise test might involve a single muscle, cyclically subjected to a prescribed extension rate, with feedback helping the subject to produce a given tension at each point of the cycle.[7] While pedal ergometers may never reach this standard, it might be worthwhile to make pedalling kinematically determined, and to prevent unnecessary reaction forces.

The design of rider support-points potentially affects pedalling efficiency, rider control, comfort, safety, etc. For example, better seats could reduce the need for hand force, reduce the need for lateral leg force (or could react it when the rider is standing, without friction and discomfort), and perhaps better react the forces arising in rapid pedalling. Steering could be reconfigured so that the hand forces of stand-up pedalling wouldn't perturb it. In view of the type of loads imposed by the rider, it should be possible to design bicycle structures more efficiently and rationally — for example, to bear the out-of-plane loads of the 'experiments' in Section 3.

While riders insist that bicycle flexibility reduces pedalling efficiency through loss of the stored elastic energy, conservation-of-energy arguments indicate that there is generally no such loss. As well, cyclists seem to be inconsistent in their perceptions and choices. (For example, they claim that lightweight racing frames are stiffer than inexpensive heavy ones; and they select extra-light frames for level, constant-speed races where mass would appear to be unimportant. Further, they fail to acknowledge that the seat and handlebars also contribute significantly to deflection, when transmitting reactions to pedalling loads.) Could some other deleterious effect of compliance account for these widespread beliefs? What tests, and what knowledge, are needed to assess the validity of such concerns? The possible disadvantages of bicycle compliance deserve to be clearly stated, and verified experimentally. With these few words the author would like to begin a dialogue on the subject, and stimulate a more critical attitude towards such popular ideas.

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[7] The elbow-tester developed by Billian and Zehalak (16) might serve in this capacity.

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