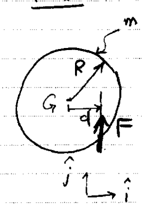
Solutions : Basant Sharma April 15 '02

① $\boxed{8.14}$

A thin flat disk floating in space. A force F applied at a distance d from the center in the y-direction.

FBD



a) what is a_G ?

LMB : $\sum F = m a_G$

$F\hat{j} = m a_G \Rightarrow \boxed{a_G = \frac{F}{m}\hat{j}}$

b) what is α ?

$AMB_{/G}$: $\sum M_{/G} = \dot{H}_{/G}$

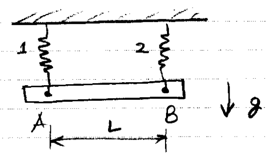
So $d\hat{i} \times F\hat{j} = I^G_{zz}\alpha\hat{k}$ where $I^G_{zz} = \frac{1}{2}mR^2$ (from table IV)

$\{F d\hat{k} = \frac{1}{2}mR^2\alpha\hat{k}\}$

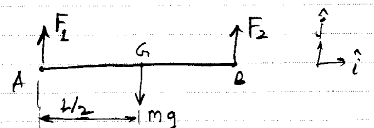
$\{\}\cdot\hat{k} \Rightarrow \boxed{\alpha = \frac{2Fd}{mR^2}}$ or $\boxed{\alpha = \frac{Fd}{mR^2}\hat{k}}$

② $\boxed{8.16}$

A uniform slender bar AB of mass m is suspended from two springs (of spring constant K). If spring 2 breaks find a) instantaneous angular acceleration of bar, b) acceleration of point A c) acceleration of point B.



a) FBD of bar (before spring 2 breaks)



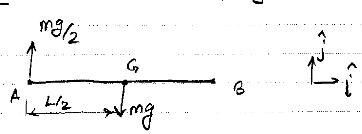
$\{LMB\}\cdot\hat{j}$: $F_1 + F_2 - mg = 0$

$\{AMB_{/G}\}\cdot\hat{k}$: $F_1 - F_2 = 0$

Since the bar is in equilibrium

So $F_1 = F_2 = \frac{mg}{2}$

FBD of bar (after spring 2 breaks)



LMB : $m a_G = -mg\hat{j} + \frac{mg}{2}\hat{j}$

So $a_G = -\frac{g}{2}\hat{j}$ ——— (1)

$AMB_{/A}$: $\dot{H}_{/G} = \sum M_{/G}$

So $I^G_{zz}\alpha\hat{k} = -\frac{L}{2}\hat{i} \times (\frac{mg}{2}\hat{j})$ ——— (2)

$(2)\cdot\hat{k}$: $\frac{mL^2}{12}\alpha = -\frac{mgL}{4}$ or $\boxed{\alpha = -\frac{3g}{L}}$ —(3)

b), c) By the kinematics of rigid bodies in plane

$a_A = a_G + a_{A/G}$

$= a_G - \omega^2 r_{A/G} + \alpha\hat{k} \times r_{A/G}$

Since at instant of spring 2 breaking $\omega = 0$, using (1) and (3) we get

$a_A = -\frac{g}{2}\hat{j} - \frac{3g}{L}\hat{k} \times (-\frac{L}{2}\hat{i})$

$= -\frac{g}{2}\hat{j} + \frac{3}{2}g\hat{j}$

or $\boxed{a_A = g\hat{j}}$ (so A accelerates up!)

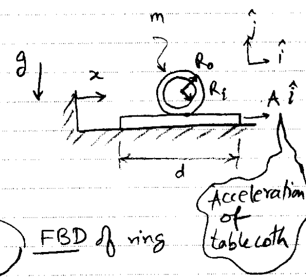
Similarly $a_B = a_G - \omega^2 r_{B/G} + \alpha\hat{k} \times r_{B/G}$

$= -\frac{g}{2}\hat{j} - \frac{3g}{L}\hat{k} \times (\frac{L}{2}\hat{i})$

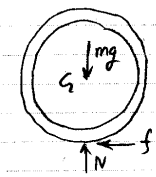
So $\boxed{a_B = -2g\hat{j}}$

③ $\boxed{8.37}$

A napkin ring lies on a table cloth and rolls w/o slip as a child pulls the table cloth w/ acceleration A. The ring starts at $x = d$.



a) what is a_G ? Rolling case FBD of ring



LMB : $\sum F = m a_G$

$-f\hat{i} + (N - mg)\hat{j} = m a_G$

where $a_G = a_G\hat{i}$ assuming the ring stays on the cloth.

$LMB\cdot\hat{i} \Rightarrow -f = m a_G$ — (1)

$AMB_{/G}$: $\{\sum M_{/G} = \dot{H}_{/G}\}\cdot\hat{k}$

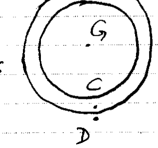
$\Rightarrow -fr = I\alpha$ —② where $I = I^G_{zz} = mR^2$

(1) and (2) give $a_G = r\alpha$, $\frac{dV_G}{dt} = r\frac{d\omega}{dt}$ —(3)

Integrating and using $V_G = 0$, $\omega = 0$ at $t = 0$:

$V_G = r\omega$ — (4) This is true for all time whether or not the ring is sliding or rolling or is on the tablecloth

No slip $\Rightarrow a_C\cdot\hat{i} = a_D\cdot\hat{i} = A$ (ie. the tangential accelerations match)



To find a_G :

$a_G = a_C + a_{G/C}$

where $a_{G/C} = -\alpha r\hat{i}$

C : pt. on ring
D : pt. on cloth
C and D coincident

Since the ring rolls on the cloth

$\Rightarrow \{a_G\hat{i} = a_C - \alpha r\hat{i}\}\cdot\hat{i}$

$\Rightarrow a_G = a_C\cdot\hat{i} - \alpha r$

$= A - \alpha r$ From slip condition above

So $a_G = A - \alpha r$ — (5)

(3) and (5) give $a_G = \frac{A}{2} \Rightarrow \boxed{a_G = \frac{A}{2}\hat{i}}$

b) When the ring rolls off the left hand end, it has moved a distance d along the tablecloth. Need to look at motion with respect to the table cloth:

$a_{G/D} = a_G - a_D$

$= \frac{A}{2}\hat{i} - A\hat{i} = -\frac{A}{2}\hat{i}$

$\Rightarrow v_{G/D} = -\frac{A}{2}t\hat{i}$, $x_{G/C} = -\frac{A}{4}t^2\hat{i}$

The ring rolls off when $|x_{G/C}| = d$

$\Rightarrow d = \frac{A}{4}t^{*2}$ or $t^* = 2\sqrt{d/A}$

Since the table cloth has acceleration $a_D = |a_D| = A \Rightarrow S = \frac{1}{2}At^2$

where $s=$ displacement of any pt. on the table cloth

So when $t = t^* = 2\sqrt{d/A}$

$$s = \frac{1}{2}A\left(\frac{4d}{A}\right) \Rightarrow \boxed{s = 2d}$$

c) When the ring hits the ground look at the velocity of the contact point:

$$\underset{\sim}{v}_C = \underset{\sim}{v}_G + \underset{\sim}{\omega} \times \underset{\sim}{r}_{C/G}$$

where $\underset{\sim}{v}_G = \frac{A}{2}t^*\hat{i} = \sqrt{Ad}\,\hat{i}$ (as $\underset{\sim}{a}_G = \frac{A}{2}\hat{i}$)

and $\underset{\sim}{\omega} = \frac{A}{2r}t^*\hat{k} = \frac{1}{r}\sqrt{Ad}\,\hat{k}$ (as $\underset{\sim}{\alpha} = \frac{A}{2r}\hat{k}$)

$$\Rightarrow \underset{\sim}{v}_C = \sqrt{Ad}\,\hat{i} + \frac{1}{r}\sqrt{Ad}\,\hat{k}\times(-r\hat{j})$$

$$= 2\sqrt{Ad}\,\hat{i}$$

Since $\underset{\sim}{v}_C \neq 0$, the ring doesnot roll at first. It will slide until $\underset{\sim}{v}_C = \underset{\sim}{0}$ or $v_G = -\omega r$.
But since (4) holds for all time, we have
$v_G = \omega r$, $v_G = -\omega r$

$$\Rightarrow \boxed{\begin{matrix} v_G = 0 \\ \omega = 0 \end{matrix}}$$

So the ring will eventually stop after sliding for sometime after it leaves the tablecloth.

d) Even if the ring slips while it is on the tablecloth, (4) still holds. Also, the transition condition from slipping to rolling still holds ($\underset{\sim}{v}_C = \underset{\sim}{0}$ or $v_G = -\omega r$). final motion

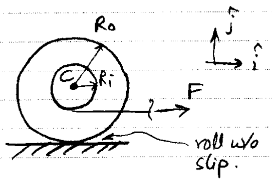
$\therefore$ $\boxed{v_G = 0,\ \omega = 0}$ just like above.

try it with an empty soda can!

④ 8.34

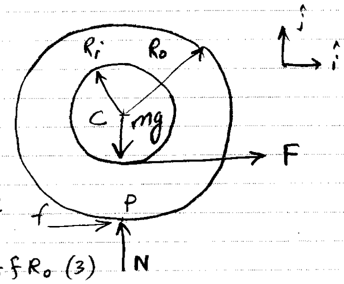
Spool rolling w/o slip and pulled by a cord.

mass of spool $= m$
Other dimensions shown →



Find: a) $\underset{\sim}{a}_C$
b) horizontal force of the ground on the spool.

FBD of spool



LMB: $\{m\underset{\sim}{a}_C = \Sigma\underset{\sim}{F}\}$

So $\{\}\cdot\hat{i}$: $ma_x = F + f$ (1)
$\{\}\cdot\hat{j}$: $0 = N - mg$ (2)

AMB$/_G$: $\{\dot{\underset{\sim}{H}}_{/G} = \Sigma\underset{\sim}{M}_{/G}\}$

$\{\}\cdot\hat{k}$: $I^G_{zz}\alpha = FR_i + fR_o$ (3)

Since the mass is "concentrated" at C $I^G_{zz} = 0$

So by (3) $FR_i + fR_o = 0$ or $f = -\frac{FR_i}{R_o}$

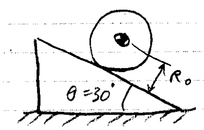
So by (1) $a_x = \frac{1}{m}\left(F - \frac{FR_i}{R_o}\right) = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)$

So a) $\boxed{a_x = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)}$

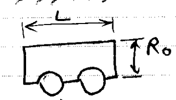
b) $\boxed{\begin{matrix} f = -\frac{FR_i}{R_o} \\ N = mg \end{matrix}}$ Net force of ground on the spool and this is towards the left

⑤ 8.47 "Race of Rollers"

Which object wins the race down the slip resistant 30° slope?

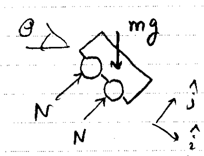


A)



$m = M_o$
Block (massless wheels, frictionless bearings)

FBD (block)

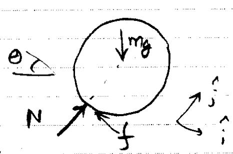


LMB: $\{\Sigma\underset{\sim}{F} = m\underset{\sim}{a}\}\cdot\hat{i}$

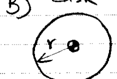
$\Rightarrow$ $mg\sin\theta = ma$

$\therefore$ $\boxed{a = g\sin\theta}$

FBD (round object: B), C), D))

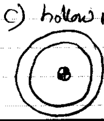


B) disk



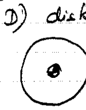
$m = M_o$
$r = 2R_o$

C) hollow pipe



$m = M_o$
$r = R_o$

D) disk



$m = 2M_o$
$r = R_o$

LMB: $\{\Sigma\underset{\sim}{F} = m\underset{\sim}{a}\}\cdot\hat{i} \Rightarrow mg\sin\theta - f = ma$ —— (1)

AMB$/_{CM}$: $\{\Sigma\underset{\sim}{M}_{/cm} - \dot{\underset{\sim}{H}}_{/CM}\}\cdot\hat{k}$

$\Rightarrow$ $-fr = -I\alpha$ where $\underset{\sim}{\alpha} = -\alpha\hat{k}$
$I = I^{cm}_{zz}$

So $f = \frac{I\alpha}{r}$ —— (2)

Rolling condition: $a = r\alpha$ (3)

(3) $\Rightarrow$ $\alpha = \frac{a}{r}$
(2) $\Rightarrow$ $f = \frac{I}{r}\left(\frac{a}{r}\right) = \frac{Ia}{r^2}$

Plugging into (1)
$mg\sin\theta - \frac{Ia}{r^2} = ma$

$$\Rightarrow \boxed{a = g\sin\theta\left(\frac{1}{1 + I/mr^2}\right)}$$

Since the accelerations are constant and the objects start at the same point the object with the largest acceleration will win.

Disk: $I = \frac{mr^2}{2} \Rightarrow a = g\sin\theta\,\frac{1}{\left(1 + \frac{\frac{1}{2}mr^2}{mr^2}\right)}$

So $a = \frac{2}{3}g\sin\theta$ for any disk.

Hollow pipe: $I = mr^2 \Rightarrow a = g\sin\theta\left(\frac{1}{1 + \frac{mr^2}{mr^2}}\right)$

So $a = \frac{1}{2}g\sin\theta$ for any pipe.

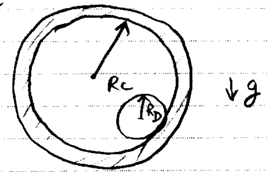
Order of finish:

1^{st}: Block (A)
(tie) 2^{nd}: All disks (B and D)
4^{th}: Pipe (C)

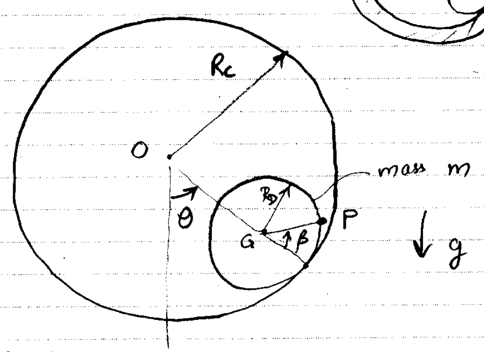
- A massless cylinder with a point mass at its centers has $I = 0$. So it will accelerate as fast as the block.
- To see an object which goes slower than the pipe, see sample 8.25 in the book. (a spool with inner radius in contact with the slope).
- Any round object with inertia will be slower than the block (the kinetic energy has both translation and rotational parts) so the block is fastest.

(6) 8.51

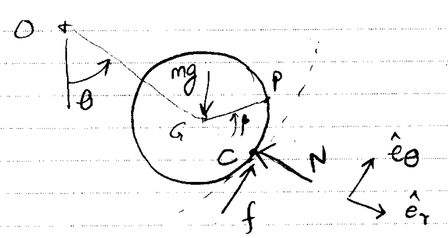
A disk rolls in a cylinder as shown w/o slip and in the presence of gravity.



a) Sketch



b) FBD of disk:



c)

LMB: $m\underline{a}_G = \sum \underline{F}$

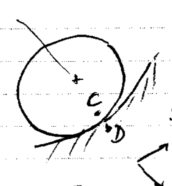
So $m(\underbrace{-r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta}_{\underline{a}_G;\ r=R_C-R_D}) = mg\cos\theta\,\hat{e}_r - mg\sin\theta\,\hat{e}_\theta - N\hat{e}_r + f\hat{e}_\theta$ —— (1)

AMB/C: $\dot{\underline{H}}_{/C} = \sum \underline{M}_{/C}$

So $\underline{r}_{G/C}\times m\underline{a}_G + I^G_{zz}(\ddot{\theta}+\ddot{\beta})\hat{k} = mgR_D\sin\theta\,\hat{k}$ —(2)

(as $\theta+\beta$ is the position of any fixed point on the disk)

d) Since C is a rolling contact



$\underline{v}_C = \underline{v}_D$ but $\underline{v}_D = \underline{0}$ (∵ the cylinder is stationary.)

So

So $\underline{v}_G + \underline{v}_{C/G} = \underline{0}$

or $\{(R_C - R_D)\dot{\theta}\hat{e}_\theta + R_D(\dot{\theta}+\dot{\beta})\hat{e}_\theta = \underline{0}\}$

picture of the disk | C, D same point

$\{\ \}\cdot\hat{e}_\theta \Rightarrow \boxed{R_C\dot{\theta} = -R_D\dot{\beta}}$

$\Rightarrow \boxed{R_C\ddot{\theta} = -R_D\ddot{\beta}}$ —(3)

e)

Using (1) in (2) and $I^G_{zz} = mR_D^2/2$ (see Table IV)

$$\Big\{-R_D\hat{e}_r\times(mg\cos\theta\,\hat{e}_r - mg\sin\theta\,\hat{e}_\theta - N\hat{e}_r + f\hat{e}_\theta) + \frac{mR_D^2}{2}(\ddot{\theta}+\ddot{\beta})\hat{k} = mgR_D\sin\theta\,\hat{k}\Big\}$$

$\{\ \}\cdot\hat{k}$ and (3) give

$$\cancel{mgR_D\sin\theta} - R_D f + \frac{mR_D^2}{2}\ddot{\theta}\left(1-\frac{R_C}{R_D}\right) = \cancel{mgR_D\sin\theta}$$

So $\frac{mR_D^2}{2}\ddot{\theta}\left(1-\frac{R_C}{R_D}\right) - R_D f = 0$ ———— (4)

from (1)$\cdot\hat{e}_\theta$: $mr\ddot{\theta} = -mg\sin\theta + f$

$\Rightarrow f = mr\ddot{\theta} + mg\sin\theta$ —— (5)

So (4) and (5) give

$$\frac{mR_D^2}{2}\left(1-\frac{R_C}{R_D}\right)\ddot{\theta} - R_D\Big[\big(m(R_C-R_D)\ddot{\theta}\big) + mg\sin\theta\Big] = 0$$

or $\frac{3}{2}(R_C-R_D)\ddot{\theta} + g\sin\theta = 0$

or $\boxed{\ddot{\theta} + \frac{2g}{3(R_C-R_D)}\sin\theta = 0}$

f) when $R_D \approx 0$

$\ddot{\theta} + \frac{2g}{3R_C}\sin\theta = 0$ (which is similar to simple pendulum)

g) Step 0: State known and unknowns.

Step 1: Draw FBDs of all bodies

Step 2: LMB and AMB equations

Step 3: Solve the equations with data and information given

Step 4: Check the answers / solutions for some "obvious" cases!