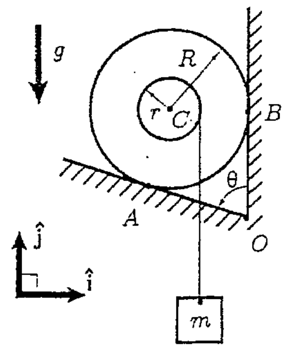
PRACTICE PROBLEMS FROM OLD PRELIMS

$k \sin\theta$

1. (35 pts.)
Statics. A massless spool of radius R is at rest in a wedge-shaped groove as shown. A mass m hangs by a massless thread wrapped around the spool axle of radius r $(0 \le r \le R)$. Wall OA is frictionless but the wall OB is frictional. There is gravity.



a.) (20 pts.) Find all reaction force <u>vectors</u> at A and B in terms of some or all of m, r, R, θ, g, $\hat{\imath}$, and $\hat{\jmath}$.

What is the smallest possible coefficient of friction μ for no motion if:

$\sin\theta$

b.) (5 pts.) $r = 0$ (justify your answer)

c.) (10 pts.) $r = R$ (justify your answer)

mass through C ∴ $B_t = 0$ so $\mu = 0$

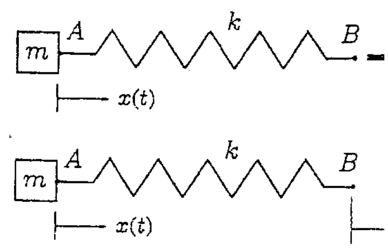
thru B ∴ A = 0 ∴ B = 0 so no μ can possibly hold

2. (35 pts.)
Spring and mass. Consider the 1-D spring-mass system shown (no gravity, no drag or friction, mass m, massless spring with constant k). In case (i), the force $F(t)$ applied at the end of spring (point B) is <u>given</u>. In case (ii), the displacement $z(t)$ of point B is <u>given</u>. When $x = z = 0$, the spring is unstretched.

x^2

(i)

(ii)



a.) (20 pts.) Using <u>power balance</u>, find an equation of motion for $x(t)$ in each of the cases (i) and (ii), in terms of the <u>given</u> quantities.

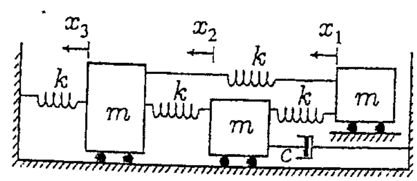
b.) (pts.) Suppose $F(t) = q\,t$ where q is a given constant and t is time. The initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$. Find $z(t)$ in terms of m, k, q, and t. (Hint: use the results for cases (i) and (ii) in part (a).)

kz

$A \sin\omega t + B\cos\omega t + \frac{q}{k}t$ Apply IC ∴ $A = B = 0$ $x = \frac{q}{k}t$ $kz = F$

$z = x = \frac{qt}{k}$??

3. (30 pts.)
Spring-mass-dashpot system. The spring-mass-dashpot system shown (all springs have constant k, all carts have mass m, the dashpot has constant c) is in equilibrium when $x_i = 0$, $i = 1, 2, 3$. Treat this as a one-dimensional system.



a.) (10 pts.) Find $\ddot{x}_2(t)$ in terms of x_i, $\dot{x}_i$, k, m, and c.

b.) (20 pts.) Let $z_1 = x_1$, $z_2 = x_2$, $z_3 = x_3$, $z_4 = \dot{x}_1$, $z_5 = \dot{x}_2$, and $z_6 = \dot{x}_3$. Find a system of first order ODE's for z_i, $i = 1, \ldots, 6$ and express it in matrix form ~~(fill in the components in the matrix equation below)~~.

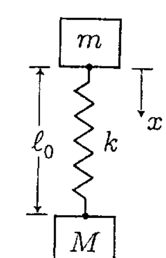
SET UP ODES SO THAT THEY CAN BE SOLVED BY MATLAB

4 (35 pts.) yes

Two blocks and a spring. Two blocks of masses M and m are connected by a spring with constant k and free length ℓ_0 that can sustain compression. Mass M is resting on the ground. There is gravity. The vertical displacement of mass m is x as shown in the figure ($x = 0$ when the spring length is ℓ_0, $x > 0$ for spring compression).

a.) (5 pts.) At what value of x is the system in equilibrium?

b.) (10 pts.) Write an equation of motion (ODE) for x, assuming M does not lift off the ground.



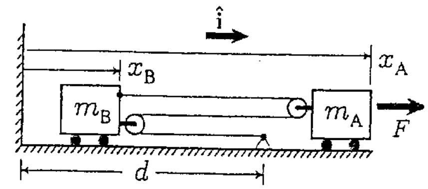
c.) (6 pts.) Draw freebody diagrams of M (i)when the spring has some general compression or extension, and (ii) when it is extended an amount so that M is about to lift off the ground.

d.) (4 pts.) What is the position x of m at the instant when M is about to lift off the ground?

(5) (30 pts.)
…ulley-Mass system. For the system shown (masses m_A, m_B, no friction, ideal …pulleys, inextensible string, force $F\hat{\imath}$) find:

a.) (15 pts.) the accelerations $\underline{a}_A$ and $\underline{a}_B$; and

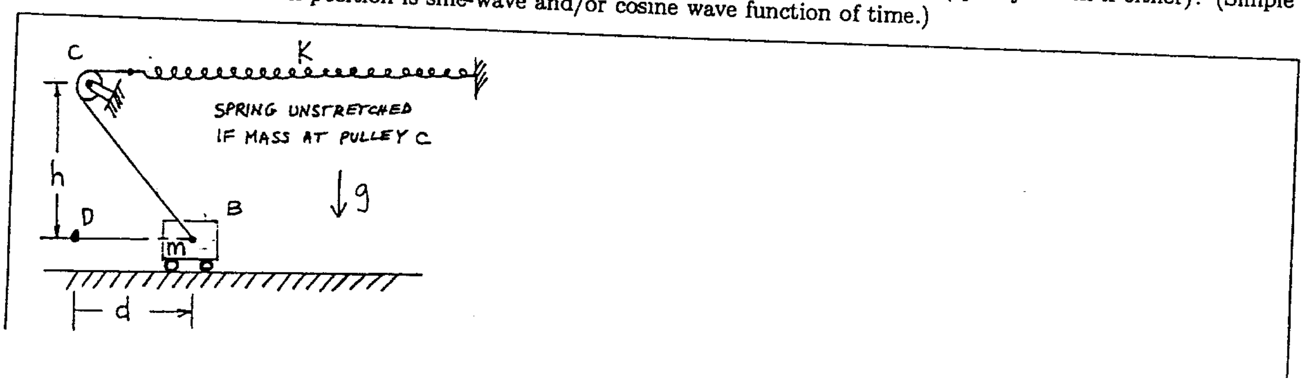
b.) (15 pts.) the tension T in the string.



(6) A cart B (mass m) rolls on a frictionless and level table. A string wrapped around a pulley at the ceiling keeps it centered in the room. The string is attached to a spring. The spring and room height are such that the spring would be relaxed if the end of the cart B was at the ceiling pulley. The ceiling height is h. The gravitational constant is g. The cart is pulled a distance d from the center of the room (D) and released.

3a) (20 points) Assuming the cart never leaves the floor, what is the speed of the cart when it passes the center of the room? (in terms of m, h, g and d).

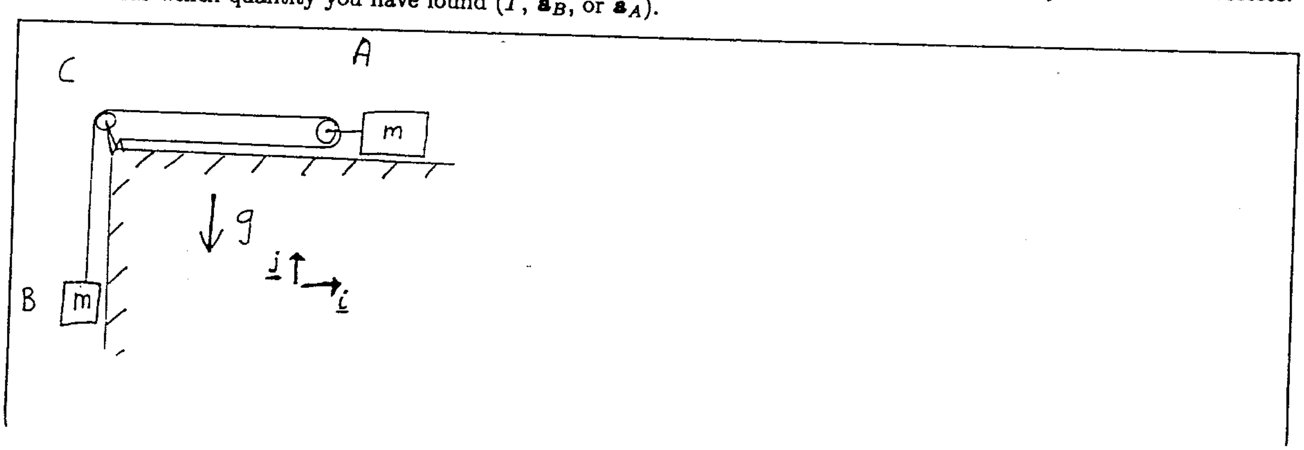
3b) (10 points) Does the cart have simple harmonic motion for small or large oscillations (specify which if either)? (Simple harmonic motion is when position is sine-wave and/or cosine wave function of time.)



(7) Two blocks both of mass m are connected to the massless pulleys as shown. There is no friction at the pulleys or where the blocks slide. The string is taught and inextensible. At the moment of interest block B is moving up with a speed of $3m/s$.

1a) (15 points) What is the velocity of block A, $\mathbf{v}_A$?

1b) (15 points) Find any of the following quantities (full credit for any one): The tension in the string (T), the acceleration of block B ($\mathbf{a}_B$), or the acceleration of block A ($\mathbf{a}_A$). Answer in terms of m and/or g and any relevant unit vectors. Make clear which quantity you have found (T, $\mathbf{a}_B$, or $\mathbf{a}_A$).



(8) 2. (30 points) A particle moves along the x-axis with an initial velocity $v_x = 60\text{m/s}$ at the origin when $t = 0$. For the first 5s it has no acceleration, and therafter it is acted upon by a retarding force which gives it a constant acceleration $a = -10\text{m/s}^2$

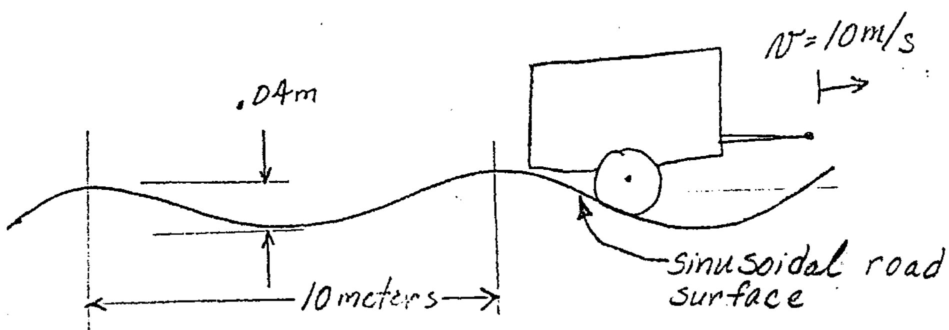
(9) (40 points) A trailer of 1000 kg mass is being towed over the bumpy road shown in the figure at a constant speed of 10 m/s. Before the trip started a 500 N weight was placed in the trailer and it was observed that the trailer moved .02 meters downward. Assume that the wheels of the trailer are always in contact with the road and that there is a linear spring between the wheels and the trailer.

(a) Model the trailer as an undamped, forced oscillator. Write the equation of motion.

(b) What is the trailer's natural frequency?

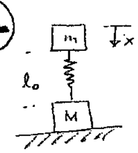
(c) Determine the amplitude of oscillation of the trailer.

(d) At what towing speed will the amplitude of oscillation be the greatest?



SOLUTIONS TO A FEW OF THEM (4, 5, 6, 7)

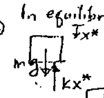
4



$\downarrow x$, from rest position

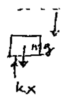
(a) In equilibrium $\downarrow x^*$

LMB $ma = 0$



$\boxed{x^* = \frac{mg}{k}}$

(b)



LMB:

$m\ddot{x} = mg - kx$

$\boxed{\ddot{x} + \frac{k}{m}x = g}$

(c) i)

general compression x

(d) For $N = 0$

$M\cancel{a_x} = \cancel{N} + Mg + kx$

$\therefore \boxed{x = -\frac{Mg}{k}}$

(e) Soln to (b) with $x = x_o, \dot{x}_o = 0$ @ $t = 0$

$x = A\cos\sqrt{\frac{k}{m}}t + B\sin\sqrt{\frac{k}{m}}t + \frac{mg}{k}$

$x_o = A + \frac{mg}{k}$

$\dot{x}_o = 0 = -A\sqrt{\ }\sin 0 + B\sqrt{\frac{k}{m}}\cos 0 \Rightarrow B = 0$

$\therefore A = (x_o - \frac{mg}{k})$

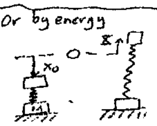
$\therefore \boxed{x = \frac{mg}{k} + (x_o - \frac{mg}{k})\cos\sqrt{\frac{k}{m}}t}$

At spring's maximum extension $\cos\sqrt{\frac{k}{m}}t = -1$

$\therefore -\frac{Mg}{k} = \frac{mg}{k} - (x_o - \frac{mg}{k})$

$\boxed{x_o = \frac{(M+2m)g}{k}}$

(e) Or by energy



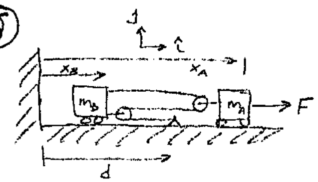
no velocity in either configuration

$\frac{1}{2}kx_o^2 - mgx_o = mgX + \frac{1}{2}kX$

where $X = -\frac{Mg}{k}$ from (d)

$\therefore x_o^2 - \frac{2mg}{k}x_o - \frac{2mg}{k}\frac{Mg}{k} - (\frac{Mg}{k})^2 = 0$

5

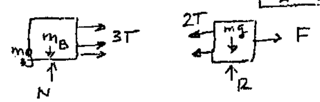


length of string $= l$

$= (d - x_B) + 2(x_A - x_B)$

$\therefore \quad l - d = 2x_A - 3x_B$

$\Rightarrow 2\ddot{x}_A - 3\ddot{x}_B = 0 \Rightarrow \boxed{a_A = \frac{3}{2}a_B}$



LMB for m_A:

$F - 2T = m_Aa_A$ —(1)

LMB for m_B:

$3T = m_Ba_B$ —(2)

$\frac{2}{3}(2) + (1) \Rightarrow F - 2T + \frac{2}{3}(3T) = \frac{2}{3}m_Ba_B + m_Aa_A$

$\therefore F = \frac{2}{3}m_B(\frac{2}{3}a_A) + m_Aa_A$

$= (\frac{4}{9}m_B + m_A)a_A$

$\boxed{a_A = \frac{F}{\frac{4}{9}m_B + m_A}}$

$\boxed{a_B = \frac{2}{3}a_A = \frac{F}{\frac{2}{3}m_B + \frac{3}{2}m_A}}$

$3T = m_Ba_B \Rightarrow$

$\boxed{T = \frac{1}{3}m_Ba_B = \frac{m_BF}{2m_B + \frac{9}{2}m_A}}$

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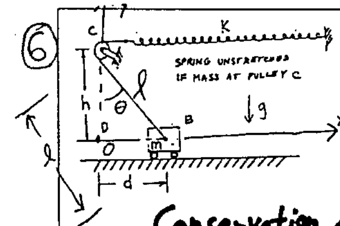


Spring geometry: (for d or X)

spring stretch $= l = \sqrt{d^2 + h^2}$

spring tension $= Fl = k\sqrt{d^2 + h^2}$

Conservation of Energy:

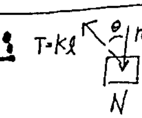
$T_{kd} + V_d = T_{kO} + V_O$

$\Rightarrow 0 + \frac{1}{2}k(d^2 + h^2) = (\frac{1}{2}mV_O^2) + (\frac{1}{2}kh^2)$

$\Rightarrow kd^2 = mV_O^2$

$\boxed{V_O = \pm\sqrt{\frac{k}{m}}d}$

FBD:



$\sum F_x = ma_x$

$-kl\sin\theta = m\ddot{X}$

$[X/l = \sin\theta]$

$\boxed{-kX = m\ddot{X}}$

Harmonic oscillator eqn., even for large X! (as well as small X)

Answer (3a) (speed at center, v_B):

at D: $V_B = \pm\sqrt{k/m}\,d$

Answer (3b) (harmonic for small/large d?):

harmonic oscill. for large d, X

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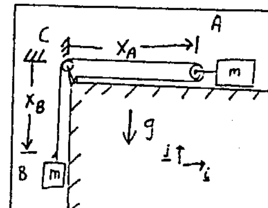

Kinematics:

$l = $ Const. $=$ string length $= 2x_A + x$

$0 = \dot{l} = 2\dot{x}_A + \dot{x}_B$ (1)

$0 = \ddot{l} = 2\ddot{x}_A + \ddot{x}_B$ (2)

(a) Given: $\underline{V}_B = \dot{x}_B(-\underline{j}) = 3\,m/s\,\underline{j}$

$\Rightarrow \dot{x}_B = -3\,m/s \Rightarrow \dot{x}_A = \frac{3}{2}$ (1)

$\Rightarrow \boxed{\underline{V}_A = \frac{3}{2}m/s\ \underline{i}}$

FBDs:

$\sum \underline{F} = m\underline{a} \Rightarrow$

$-T + mg = +m\ddot{x}_B$ (3)

$\sum \underline{F} = m\underline{a} \Rightarrow -2T = m\ddot{x}_A$ (4)

Algebra:

$2 \times (4) + (3) \Rightarrow mg = 5T \Rightarrow \boxed{T = \frac{mg}{5}}$

(4) $\Rightarrow \ddot{x}_A = -(2/5)\,mg/m$, $\underline{a}_A = \ddot{x}_A\underline{i}$

(3) $\Rightarrow \ddot{x}_B = (4/5)\,mg/m$, $\underline{a}_B = -\ddot{x}_B\underline{j}$

Answer (a) (v_A):

$\underline{V}_A = \frac{3}{2}\,m/s\ \underline{i}$

Answer (b) (T, a_A, or a_B):

$T = \frac{mg}{5}$	$\underline{a}_A = \frac{-2}{5}g\underline{i}$	$\underline{a}_B = \frac{-4}{5}g$