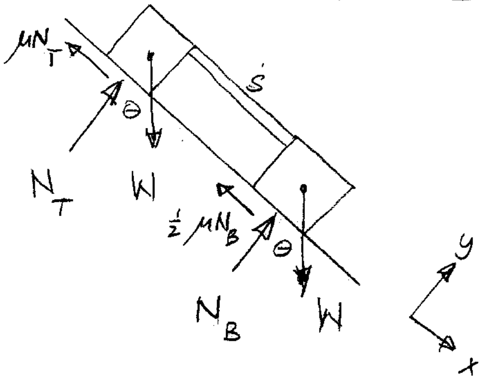
SOLUTION : PROB 1 (a) SOLNS - FINAL 12/99



We have $\underline{M}_G = \dot{\underline{H}}_G = 0$ about the CM of the two blocks. We neglect the moments produced by μN and by any distribution of forces across the bottom of each block, since the distance s between the blocks is so much larger than any block dimension.

in $\hat{k}$

$$\Sigma M_G = \frac{s}{2} N_B - \frac{s}{2} N_T = 0 \implies \underline{N_B = N_T}$$

$$\Sigma F_y = m a_y = 0 \implies 2W\cos\theta = N_B + N_T = 2N$$

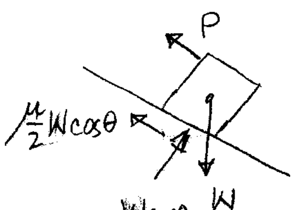
or $\underline{N = W\cos\theta}$

$$\Sigma F_x = m\ddot{x} \implies 2W\sin\theta - \mu N_T - \frac{\mu}{2} N_B = \left(\frac{W}{g} + \frac{W}{g}\right)\ddot{x}$$

$$2\cancel{W}\sin\theta - \frac{3}{2}\mu \cancel{W}\cos\theta = \frac{2\cancel{W}}{g}\ddot{x}$$

$$\therefore \boxed{\ddot{x} = g\left(\sin\theta - \frac{3}{4}\mu\cos\theta\right)}$$

(b) To find the internal force, look at the motion of one of the blocks.



$$W\sin\theta - \frac{\mu}{2} W\cos\theta - P = \frac{W}{g}\ddot{x}$$

$$\therefore P = W\left(\sin\theta - \frac{\mu}{2}\cos\theta - \frac{\ddot{x}}{g}\right)$$

and substituting from (a) for $\ddot{x}$

$$P = W\left(\cancel{\sin\theta} - \frac{\mu}{2}\cos\theta - \left[\cancel{\sin\theta} - \frac{3}{4}\mu\cos\theta\right]\right)$$

$$\boxed{P = \frac{\mu W}{4}\cos\theta} \quad \text{tension}$$

(c) From (a) the acceleration of the system is constant. Hence $v^2 - \cancel{v_0^2}^{\,0} = 2ad$

$$v = \sqrt{2ad} = \boxed{\sqrt{2dg\left(\sin\theta - \frac{3}{4}\mu\cos\theta\right)} = v}$$

This result could also be found by using <u>work-energy principle</u>:

$$\Delta KE = \text{Work done}$$

$$\frac{1}{2}(2m)v^2 - \cancel{\frac{1}{2}(2m)v_0^2}^{\,0} = \int \underline{F}\cdot d\underline{s} = F_x d = \left(2W\sin\theta - \frac{3}{2}\mu W\cos\theta\right)d$$

$$\text{or } v = \sqrt{2dg\left(\sin\theta - \frac{3}{4}\mu\cos\theta\right)}$$

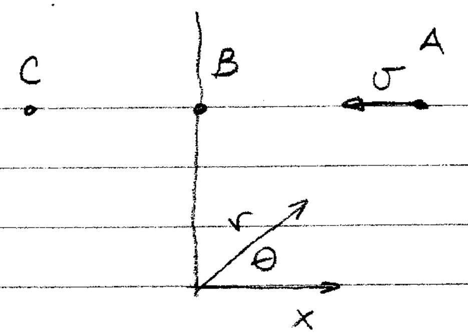
SOLN: PROB (1d)

(d) i.) If the blocks were interchanged, the free body diagram would the same insofar as x forces go $\Rightarrow$ acceleration and end velocity are the same

the rod force becomes compression since top block wants to move faster than bottom, but magnitude is same.

ii.) Changing from a rod to a string only changes the internal forces. Since only external forces affect the motion of the mass center, the acceleration and final velocity of the mass center are unchanged. The string cannot support a force.

(2)



<u>At A</u>

$r > 0$, $\dot{r} < 0$, $\ddot{r} < 0$

$\theta > 0$, $\dot{\theta} > 0$, $\ddot{\theta} > 0$

<u>At B</u>

$r > 0$, $\dot{r} = 0$, $\ddot{r} > 0$

$\theta > 0$, $\dot{\theta} > 0$, $\ddot{\theta} = 0$

<u>At C</u> $r > 0$, $\dot{r} > 0$, $\ddot{r} > 0$

$\theta > 0$, $\dot{\theta} > 0$, $\ddot{\theta} < 0$

<u>at B</u>

$\underline{v} = v\,\hat{e}_\theta$

$\underline{a} = 0$

③ ⓐ Your tax dollars at work! The definitions are all wrong or incomplete

i.) Angular momentum $\underline{H}_o = \underline{r} \times m\underline{v}$ for point mass

or $\underline{\underline{I}} \cdot \underline{\omega}$

Listed defn doesn't even have correct units!

Spinning bumper cars conserve angular momentum as passengers move arms.

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ii.) Centripetal force is a commonly used term to express the force when particles move in circles $\{ -m\underline{a}$ where $\underline{a}$ is a centripetal accel $-r\omega^2$

It technically is not a force but the common defn would even apply for non-circular motion — anytime an acc exists

People are flattened against walls of spinning ride.

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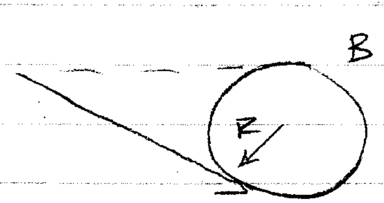
iii) $\underline{F} = m\underline{a}$ tension, electric, contact ...

Defn is wrong because forces only do work (change energy) if motion is in direction of force.

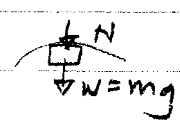
Operation of any ride that accelerates.

ⓑ



If frictionless, speed at top of arc = speed at start

FBD when cart at B. Wheels will be in contact if $N > 0$,

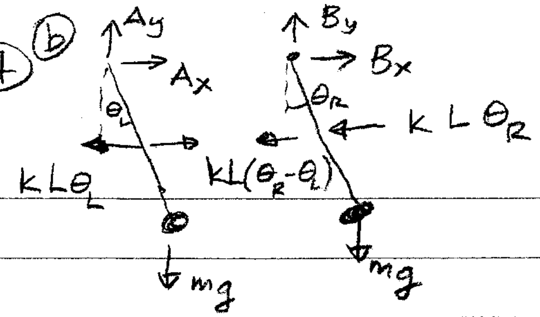


Circular motion

$$N + mg = \frac{mv^2}{R}$$

$$v^2 > gR$$

④ (b)



$\sin\theta \approx \theta$ for small angles

LIKE LAB 2, except pendula

ⓐ 2 degrees of freedom

∴ 2 normal modes

ⓒ $\Sigma M_A = m(2L)^2\ddot{\theta}_L = L[kL(\theta_R-\theta_L) - kL\theta_L] - mg(L\theta_L)$

$$\ddot{\theta}_L = \frac{kL^2}{4mL^2}(\theta_R - 2\theta_L) - \frac{mgL}{4mL^2}\theta_L$$

$$\ddot{\theta}_L = -\frac{k}{2m}\theta_L - \frac{g}{4L}\theta_L + \frac{k}{4m}\theta_R$$

$$\Sigma M_B = m(2L)^2\ddot{\theta}_R = L[-kL(\theta_R-\theta_L) - kL\theta_R] - mgL\theta_R$$

$$\ddot{\theta}_R = -\frac{k}{2m}\theta_R - \frac{g}{4L}\theta_R + \frac{k}{4m}\theta_L$$

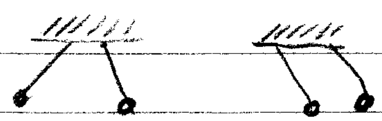
$$\therefore \begin{pmatrix}\ddot{\theta}_L \\ \ddot{\theta}_R\end{pmatrix} = \begin{pmatrix} -\frac{k}{2m}-\frac{g}{4L} & \frac{k}{4m} \\ \frac{k}{4m} & -\frac{k}{2m}+\frac{g}{4L}\end{pmatrix}\begin{pmatrix}\theta_L \\ \theta_R\end{pmatrix}$$

ⓓ Need eigenvalues

$$\begin{pmatrix} \frac{k}{2m}+\frac{g}{4L}-\omega^2 & -\frac{k}{4m} \\ -\frac{k}{4m} & \frac{k}{2m}+\frac{g}{4L}-\omega^2 \end{pmatrix} = 0$$

$$\left(\frac{k}{2m}+\frac{g}{4L}\right)^2 - 2\omega^2\left(\frac{k}{2m}+\frac{g}{4L}\right) + \omega^4 - \frac{k^2}{4^2m^2} = 0$$

$$\left[\omega^2 - \left(\frac{k}{4m}+\frac{g}{2L}\right)\right]\left(\omega^2 - \left[\frac{g}{2L}+\frac{k}{2m}\right]\right) = 0$$

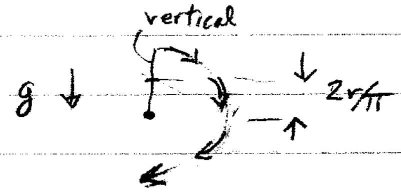
ⓔ 

in each case Restoring Moment ~ θ_L, θ_R and each bob has same freq.

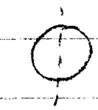
5 ⓐ

Energy is conserved

$$mg\Delta h = \Delta KE$$



$$2mg\left(\frac{2r}{\pi}\right) = \frac{1}{2} I_{xx} \Omega_{bottom}^2$$



$I_{xx} = \frac{1}{2} m r^2$ for full ring but also for ½ ring since ½ mass

$$\cancel{m}\frac{4gr}{\pi} = \frac{1}{2}\left(\frac{1}{2}\cancel{m}r^2\right)\Omega_{bottom}^2$$

$$\Omega_{bot} = 4\sqrt{\frac{g}{\pi r}}$$

$$\boxed{v_{CM} = \Omega \frac{2r}{\pi} = 8\sqrt{gr/\pi^3}}$$

ⓑ $\underline{\omega}_{TOTAL} = \underline{\Omega}_{bottom} + \underline{\omega} = \boxed{4\sqrt{\frac{g}{\pi r}}\ \hat{\imath} + \omega\hat{k}}$

Assumes falls to left

ⓒ $\underline{\alpha} = \frac{d}{dt}\left(\underline{\Omega}_{bottom} + \underline{\omega}\right) + \underline{\omega} \times \underline{\Omega}_{bottom}$

max at bottom   const

$$\therefore \boxed{\underline{\alpha} = 4\omega\sqrt{\frac{g}{\pi r}}\ \hat{\jmath}}$$

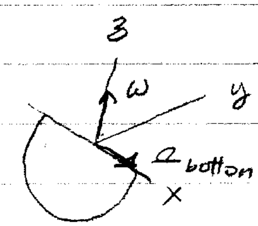
ⓓ

$$\underline{\underline{I}}_o = \begin{pmatrix} \frac{1}{2}mr^2 & 0 & 0 \\ 0 & mr^2 & 0 \\ 0 & 0 & \frac{1}{2}mr^2 \end{pmatrix}$$

$I_{xz} = 0$ by symmetry

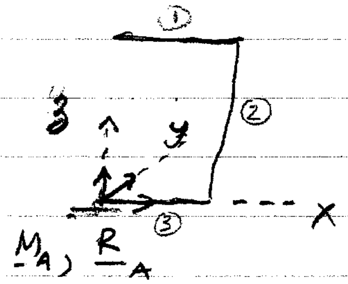
I_{xy} & $I_{yz} = 0$ since $y = 0$

ⓔ $\underline{H}_o = \underline{\underline{I}}_o \cdot \underline{\omega}_{TOTAL}$



$$= \frac{1}{2}mr^2\left(\Omega_b\ \hat{\imath} + \omega\hat{k}\right)$$

⑥ Planar rotation about a single axis



circular motion

$$\underline{F} = \sum m\,\underline{a} = -2m\left(\frac{L}{2}\right)\omega^2\,\hat{\imath} - mL\omega^2\,\hat{\imath} + 2m\left(\frac{L}{2}\right)\alpha\,\hat{\jmath} + mL\alpha\,\hat{\jmath}$$

$$\boxed{\underline{R}_A = -2mL\omega^2\,\hat{\imath} + 2mL\alpha\,\hat{\jmath}}$$

$$\underline{M} = \left(-I_{x3}\,\dot{\omega}_3 + I_{y3}\,\omega_3^2\right)\hat{\imath} - \left(I_{y3}\,\dot{\omega}_3 + I_{x3}\,\omega_3^2\right)\hat{\jmath} + I_{33}\,\dot{\omega}_3\,\hat{k}$$

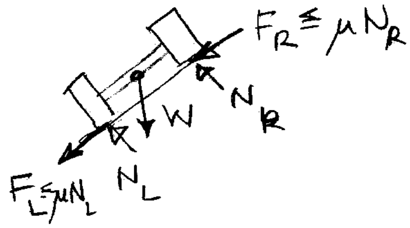
$$I_{33} = \underbrace{\frac{1}{3}(2m)L^2}_{①+③} + \underbrace{mL^2}_{②} = \frac{5}{3}mL^2$$

$$I_{x3} = \underbrace{0}_{③} + \underbrace{\int_0^L x\,L\,dm}_{①} + \underbrace{\int_0^L L\,z\,dm}_{②} = Lm\left(\frac{L}{2} + \frac{L}{2}\right) = mL^2$$

$$I_{y3} = 0$$

$$\therefore \boxed{\underline{M} = -mL^2\alpha\,\hat{\imath} - mL^2\omega^2\,\hat{\jmath} + \frac{5}{3}mL^2\alpha\,\hat{k}}$$

(a)



The reaction forces acting on the auto's tires have a component pointing toward the center of curvature (as necessary to produce the observed centripetal acceleration). The frictional forces too may point in part toward the center of curvature; whether they point up or down depends on θ (large θ, small v^2/R & they will point up the slope to oppose the sliding down). The values of the four unknown forces come from $\underline{F} = m\underline{a}$, $\underline{M} = \dot{\underline{H}} = 0$ (ignoring gyroscopic moments of wheels)

(b) There are six things that must be controlled

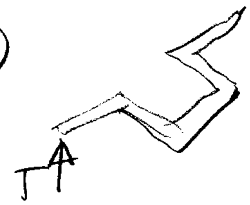
3 linear accelerations

3 angular accelerations

Thrusters only operate in one direction $\therefore$ need 12

5 cont'd

(c)



Only impulsive forces need be considered

i.) Yes. No impulsive moment about T $\Rightarrow$ $\underline{H}_T$ = const.

ii.) No. Impact force at T has moment

iii.) No. Rotation axis changes

iv.) No. Energy is lost to noise, elastic waves, heat, etc.

v.) No. External impulse acts.

(d) The amount of rain that strikes your head depends on the time you're in the rain & the relative vertical velocity

$\therefore$ run to minimize

The amount that strikes your side depends on relative horiz vel (i.e., your speed) x time out ($\sim \frac{1}{vel}$)

$\therefore$ independent of whether you walk or run

(e) $\underline{H} \parallel \underline{\omega} \Rightarrow$ pure spin

$\underline{H} \nparallel \underline{\omega} \Rightarrow$ wobble