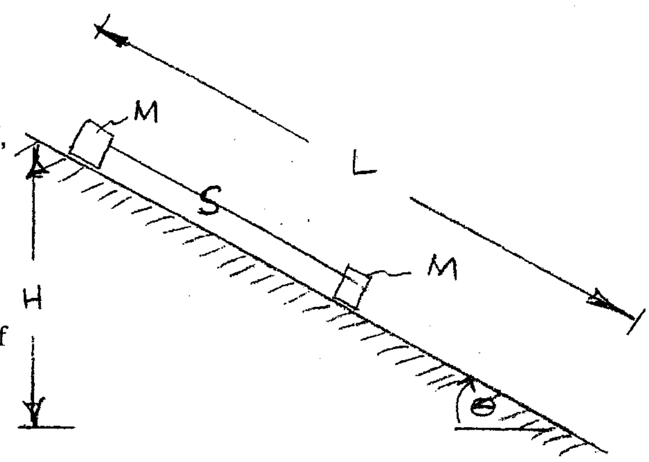
OLD FINALS QUESTIONS (SOLUTIONS NOT AVAILABLE) TEAM 203

1. (16 points) Two blocks, each of mass M, are connected across their tops by a massless rigid rod of length S; the blocks' dimensions are small compared to S. The blocks slide down a slope of length L and height H. The coefficient of dynamic friction on the bottom block is $\mu/2$ while the top block slides with μ.



a). (5 pts). What is the acceleration of the center of mass of the two blocks just after they start from rest?

b). (4 pts). What is the force in the rod?

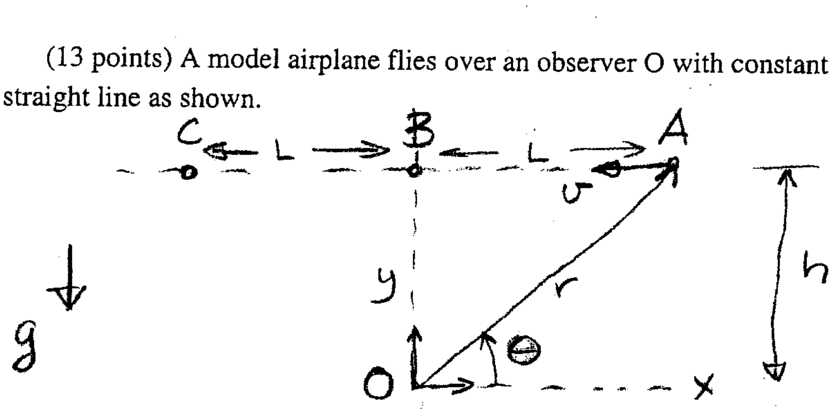
c). (3 pts). What is the speed of the center of mass for the blocks after they have traveled a distance d down the slope, having started from rest?

d). (4 pts). How would your answers to the above questions differ in the following two variations? Qualitative responses to d) will suffice.

i) The two blocks are interchanged so that the slippery one is on the top; and

ii) The rigid rod is replaced by a massless string.

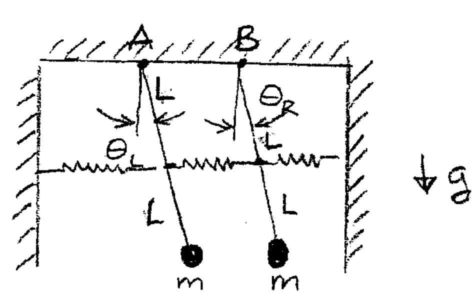
2. (13 points) A model airplane flies over an observer O with constant speed v in a straight line as shown.



a) (9 pts) Determine the signs (plus, minus or zero) for r, r', r'', θ, θ', and θ'' for each position.

b) (4 pts) Write the model's velocity and acceleration in polar coordinates at B.

3. (20 points). Two particles of mass m lie on rigid, massless rods of length $2L$ hinged about A and B. Three equal springs with stiffness k are attached to the midpoints of the rods as shown; the springs can be considered to always remain horizontal. Assume small oscillations. Neglect gravity and friction.



a) (2 pts.) How many degrees of freedom does the system have? How many normal modes exist?

b) (4 pts.) Draw free-body diagrams for the pendula when they are at angles θ_L and θ_R.

c) (6 pts.) Write the equations of motion for θ_L and θ_R. Leave your answer in the form **x'' = [A]x**.

d) (5 pts.) What are the normal mode frequencies?

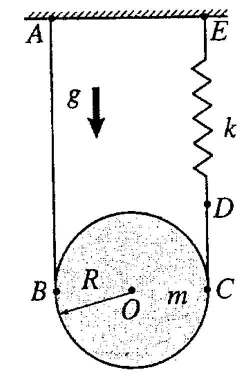
e) (3 pts.) What mode shape corresponds to each frequency? Justify your answer, arguing why each of these mode shapes leads to simple harmonic motion.

4. (20 pts.)
String, uniform disk, and spring. Inextensible string $ABCD$ is wrapped around the uniform disk (mass m, radius R) and connected to spring DE (constant k, unstretched length ℓ_0). There is gravity.

a.) (5 pts.) Find the total length ℓ_{eq} of the spring when the system is in equilibrium.

b.) (5 pts.) Assuming no slip between disk and string, let x be the downward displacement of point O from its equilibrium position. Find a relation between the spring length $\ell(t)$ and $x(t)$ at time t involving some or all of m, k, ℓ_0, ℓ_{eq}, R, and g.

c.) (10 pts.) Find an ODE for $x(t)$ involving some or all of m, k, ℓ_0, ℓ_{eq}, R, and g.



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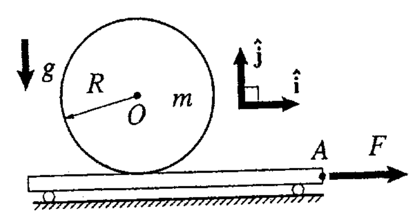
5. (20 pts.)
Uniform disk on a moving slab. A uniform disk of mass m and radius R is sitting on a slab of mass m that is resting on frictionless, massless, rollers. At time $t = 0$, a constant force $F\hat{\imath}$ is applied to the slab. Assume no slip between the disk and slab. There is gravity. Just after the application of the force, find the following in terms of some or all of g, m, R, F, $\hat{\imath}$, and $\hat{\jmath}$:

a.) (10 pts.) the acceleration vector $\mathbf{a}_O$ (O is the disk c.o.m.)

b.) (5 pts.) the acceleration vector $\mathbf{a}_A$ (A is a point as shown on the slab)

c.) (5 pts.) the largest possible value of F (in terms of μ, m, g, and R) for no slip between disk and slab (μ is the friction coefficient between the slab and disk.)



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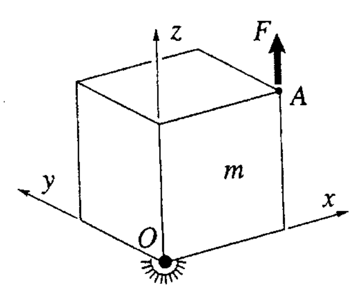
6. (20 pts.)
Uniform cube attached to ball-and-socket joint. A uniform cube of mass m and side of length ℓ is at rest until a force $\mathbf{F} = F\hat{\mathbf{k}}$ is suddenly applied at point A at $t = 0$. Point O on the cube is held fixed by a frictionless ball-and-socket joint. There is no gravity. Find $\mathbf{a}_A$, the acceleration of point A, just after $t = 0$, in terms of m, ℓ, F, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{\mathbf{k}}$.

[Useful identity: $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.]



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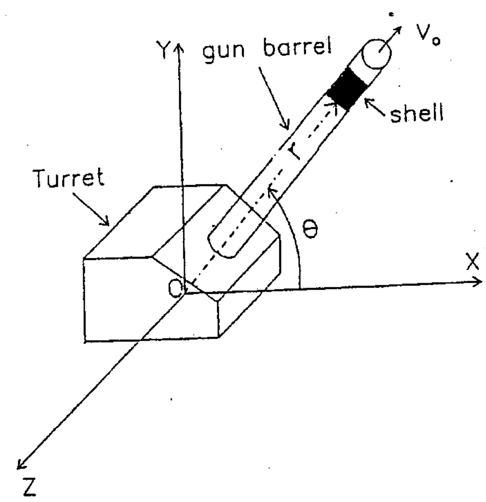
7. (20 pts.)
Shell, gun barrel, and turret. The shell (point mass m) is moving with constant speed v_0 relative to the gun barrel. Its distance from the origin is r. The turret is rotating about the y-axis with angular speed ω_1=constant while the barrel is rising by rotating about the z-axis with $\dot{\theta} = \omega_2$=constant. At the instant of interest, the barrel is in the xy-plane and $\theta = \frac{\pi}{4}$. At that instant, find:

a.) (5 pts.) the absolute angular velocity vector ω of the barrel in terms of some or all of the quantities given above (v_0, r, m, ω_1, ω_2, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{\mathbf{k}}$)

b.) (5 pts.) the shell's absolute velocity vector $\mathbf{v}$ in terms of the given quantities.

c.) (10 pts.) the total force vector $\mathbf{F}$ acting on the shell in terms of the given quantities



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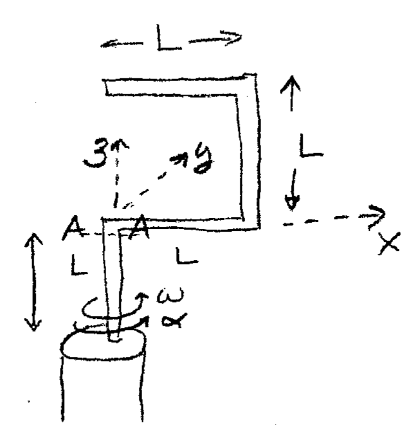
8. (16 points). A structure, made of 4 rods (each of mass m and length L), spins around a vertical shaft with an instantaneous spin rate ω and an angular acceleration α. At the moment shown determine the forces (axial and shear) and moments (bending and twisting) that act at section AA of the structure. Ignore gravity and the mass of the right-angle corners.



...EMENT PARK SCIENCE KEY TERMS

Angular Momentum is the velocity in degrees of a circle multiplied by mass.

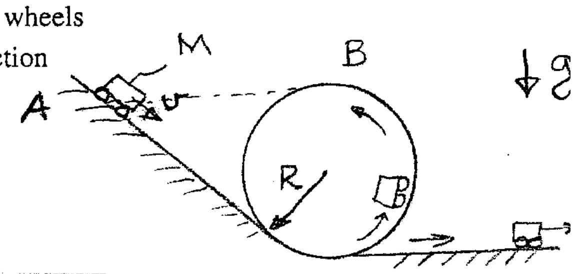
Centripetal Force is a force that holds an object spinning in a circle.

Force is the push or the pull that gives an object energy.

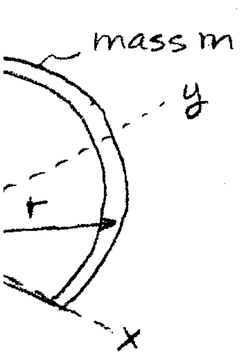
a) (9 pts). Reproduced alongside is a section of a flyer from an exhibit currently at the Ithaca Sciencenter, "*Amusement Park Science*". This exhibit, produced by the Discovery Center Museum in Rockford, Illinois, and funded by the NSF, has a series of models and demonstrations based on amusement park rides to illustrate dynamical concepts. Give your own definitions for the three terms listed* ~~and use each term in a sentence describing how a specific amusement park ride (e.g., roller coaster or ferris wheel) works.~~

* State any errors in the given definitio

...nd the minimum speed at A that a roller ...ust be moving at in order that its wheels ...ct with the tracks at B. Ignore friction ...ravity.

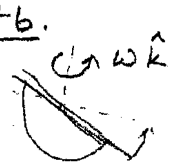


...). A semicircular ring of mass m and radius r pivots frictionlessly about a ...xle but is also able to rotate simultaneously about the vertical z axis. The ...ss lies at distance $2r/\pi$ off the diameter.



part a.

ring starts in xz plane & falls, pivoting around x a



as ring is falling, the horizontal axle rotates with ω k̂

...s.) If the ring is released from rest in a vertical position and falls throug a ...ity field g, what is the maximum speed of the center of mass?

...ts.) If the ring is simultaneously spun with angular velocity ω about the z axis,