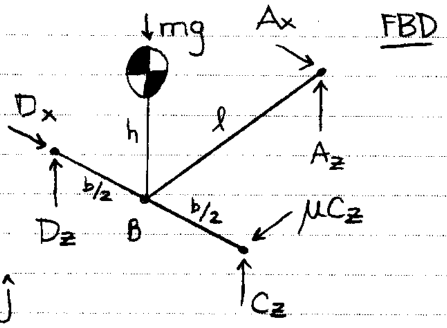
6.74 A branch gets caught in the right rear wheel of a tricycle causing the wheel to skid in the $\hat{j}$-direction. The wheels at A & D roll without slip and are massless. What's the acceleration of the tricycle?

- A & D roll w/o slip in $\hat{j}$-direction
- wheel at C skids in $\hat{j}$-direction



LMB : $\sum \underset{\sim}{F} = m\underset{\sim}{a} = ma\hat{j}$

LMB $\cdot \hat{j} \Rightarrow \; -\mu C_z = ma \quad (1)$

$AMB_{/B} \cdot \hat{j} : \{\sum \underset{\sim}{M}_{/B} = \dot{\underset{\sim}{H}}_{/B}\} \cdot \hat{j}$ — moments about axis through B // to $\hat{j}$-direction

$\Rightarrow \; -C_z\left(\frac{b}{2}\right) + D_z\left(\frac{b}{2}\right) = 0$

$\therefore \; C_z = D_z \quad (2)$

$AMB_{/A} \cdot \hat{i} : \{\sum \underset{\sim}{M}_{/A} = \dot{\underset{\sim}{H}}_{/A}\} \cdot \hat{i}$ — moments about axis through A // to $\hat{i}$-direction

$\Rightarrow -C_z(\ell) - D_z(\ell) + mg(\ell) = -mah$

$mg - (C_z + D_z) = -ma\frac{h}{\ell} \quad (3)$

(2) into (3): $mg - 2C_z = -ma\frac{h}{\ell} \quad (4)$

(1) into (4): $\cancel{m}g + \frac{2\cancel{m}a}{\mu} = -\cancel{m}a\frac{h}{\ell}$

$$a\left(\frac{2}{\mu} + \frac{h}{\ell}\right) = -g$$

$$\Rightarrow a = \frac{-g}{\frac{2}{\mu} + \frac{h}{\ell}} = \frac{-\frac{\mu g}{2}}{1 + \frac{\mu h}{2\ell}}$$

$$\boxed{\therefore \; \underset{\sim}{a} = -\frac{\mu g}{2}\left(\frac{1}{1 + \frac{\mu h}{2\ell}}\right)\hat{j}}$$

Checks: ① When $\mu = 0$, $\underset{\sim}{a} = \underset{\sim}{0}$ (no forces in $\hat{j}$-direction $\Rightarrow$ constant $\underset{\sim}{v}$)

② When $h = 0$, $\underset{\sim}{a} = -\frac{\mu g}{2}\hat{j}$ (Half of the rider's weight is supported at wheel C.)

position: $\underset{\sim}{r}(t) = R\cos(\dot{\theta}t)\hat{\imath} + R\sin(\dot{\theta}t)\hat{\jmath}$

where $\dot{\theta} = \text{constant}$

a) $\underset{\sim}{v} = \dot{\underset{\sim}{r}} = R\dot{\theta}\left(-\sin(\dot{\theta}t)\hat{\imath} + \cos(\dot{\theta}t)\hat{\jmath}\right)$

$\therefore \underset{\sim}{v} \neq \underset{\sim}{0}$ since the velocity depends on time

b) $\underset{\sim}{v} \neq$ constant (see a))

c) $|\underset{\sim}{v}| = R\dot{\theta} =$ constant since $\dot{\theta} =$ const.

d) $\underset{\sim}{a} = \dot{\underset{\sim}{v}} = -R\dot{\theta}^2\left(\cos(\dot{\theta}t)\hat{\imath} + \sin(\dot{\theta}t)\hat{\jmath}\right)$

$\therefore \underset{\sim}{a} \neq \underset{\sim}{0}$ since the acceleration depends on time

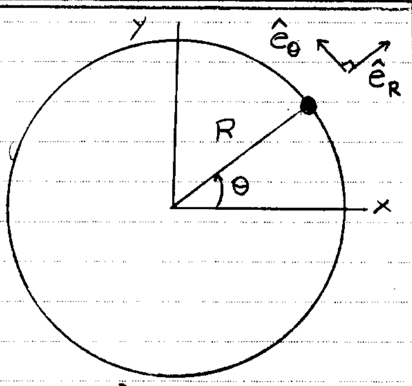
e) $\underset{\sim}{a} \neq$ constant (see d))

f) $|\underset{\sim}{a}| = R\dot{\theta}^2 =$ constant since $\dot{\theta} =$ const.

g) $\underset{\sim}{v} \cdot \underset{\sim}{a} = -R^2\dot{\theta}^3\left(-\sin(\dot{\theta}t)\cos(\dot{\theta}t) + \cos(\dot{\theta}t)\sin(\dot{\theta}t)\right) = \underline{\underline{0}}$

$\therefore \quad \underline{\underline{\underset{\sim}{v} \perp \underset{\sim}{a}}}$

<u>7.16</u>



given

$\theta = \theta_0 \cos\lambda t$

$\Rightarrow \quad \dot{\theta} = -\theta_0\lambda\sin\lambda t$

$\ddot{\theta} = -\theta_0\lambda^2\cos\lambda t$

$\underset{\sim}{a} = -R\dot{\theta}^2\hat{e}_R + R\ddot{\theta}\hat{e}_\theta$

$$\left.\begin{aligned} \text{where } \dot{\theta}^2 &= \theta_0^2\lambda^2\sin^2\lambda t \\ &= \lambda^2(\theta_0^2 - \theta_0^2\cos^2\lambda t) \\ &= \lambda^2(\theta_0^2 - \theta^2) \\ \ddot{\theta} &= -\lambda^2\theta \end{aligned}\right\} \text{get } |\underset{\sim}{a}| \text{ in terms of } \theta$$

$\Rightarrow |\underset{\sim}{a}|^2 = R^2\dot{\theta}^4 + R^2\ddot{\theta}^2$

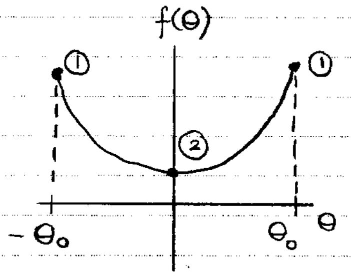
$|\underset{\sim}{a}|^2 = R^2\left[\lambda^4(\theta_0^2 - \theta^2)^2 + \lambda^4\theta^2\right]$

$|\underset{\sim}{a}|^2(\theta) = R^2\lambda^4\underbrace{\left[(\theta_0^2 - \theta^2)^2 + \theta^2\right]}_{f(\theta)}$

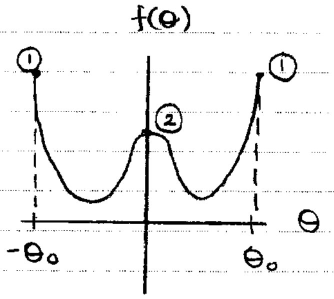
(continued)

Maximum acceleration occurs at the maximum value of

$$f(\theta) = (\theta_o^2 - \theta^2)^2 + \theta^2 = \theta^4 + \theta^2(1 - 2\theta_o^2) + \theta_o^4$$



$\theta_o < \frac{1}{2}$



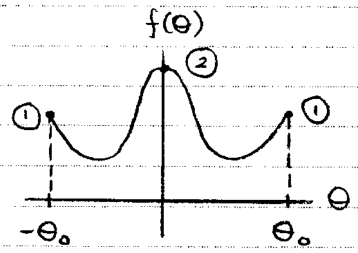
$\frac{1}{2} < \theta_o < 1$

① $f(\theta = |\theta_o|) = \theta_o^2$

② $f(\theta = 0) = \theta_o^4$

∴ Max value of $|\underset{\sim}{a}|$ will be at $\theta = 0$ only when $\underline{\theta_o \geq 1}$.

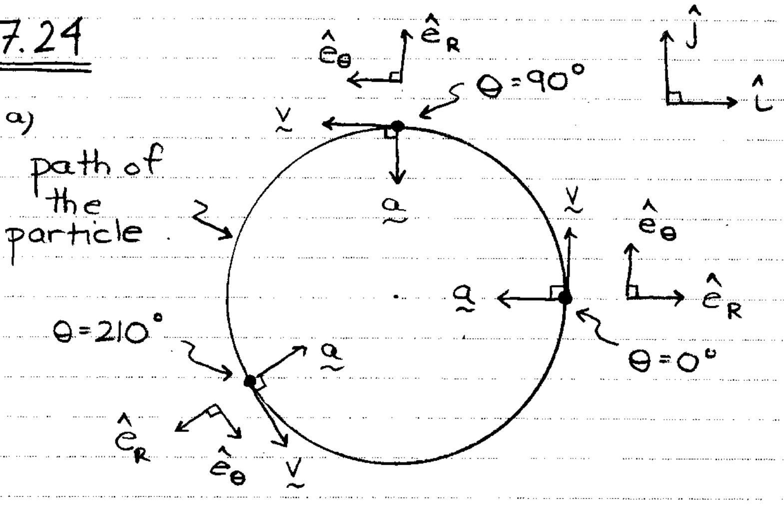
$\Rightarrow \boxed{|\underset{\sim}{a}| = R\lambda^2\theta_o^2}$



$\theta_o > 1$

7.24

a)



b) $\boxed{\dot{\theta} = \frac{2\pi}{\tau} = \text{constant}}$

@ $\underline{\theta = 0^\circ}$: $\hat{e}_R = \hat{\imath}$, $\hat{e}_\theta = \hat{\jmath}$

$\underset{\sim}{v} = R\dot{\theta}\hat{\jmath}$, $\underset{\sim}{a} = -R\dot{\theta}^2\hat{\imath}$

@ $\underline{\theta = 90^\circ}$: $\hat{e}_R = \hat{\jmath}$, $\hat{e}_\theta = -\hat{\imath}$

$\underset{\sim}{v} = -R\dot{\theta}\hat{\imath}$, $\underset{\sim}{a} = -R\dot{\theta}^2\hat{\jmath}$

(continued)

7.24 (cont'd)

@ $\theta = 210°$: $\hat{e}_R = -\frac{\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}$

$\hat{e}_\theta = \frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}$

$\underset{\sim}{v} = R\dot{\theta}\left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$

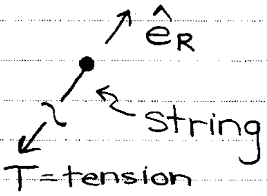
$\underset{\sim}{a} = R\dot{\theta}^2\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$

c) LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$\{-T\hat{e}_R = m(-R\dot{\theta}^2\hat{e}_R)\}$

$\{\}\cdot\hat{e}_R \Rightarrow \boxed{T = mR\dot{\theta}^2}$

FBD



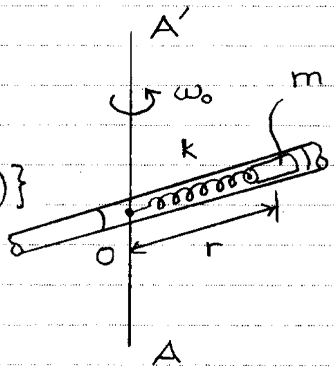
d) Since $\dot{\theta}$ = constant, there is no tangential acceleration. Therefore, the radial tension is all that's needed to keep the motion going.

e) $\underset{\sim}{T} = -T\hat{e}_R = T\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$ @ $\theta = 210°$

$\therefore \boxed{F_x = \frac{\sqrt{3}}{2}T = \frac{\sqrt{3}}{2}mR\dot{\theta}^2}$

7.29

LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$



$\{-k(r-r_o)\hat{e}_R = m(-r\omega_o^2\hat{e}_r)\}$

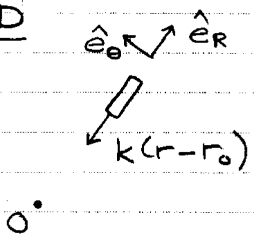
$\{\}\cdot\hat{e}_R \Rightarrow -k(r-r_o) = -mr\omega_o^2$

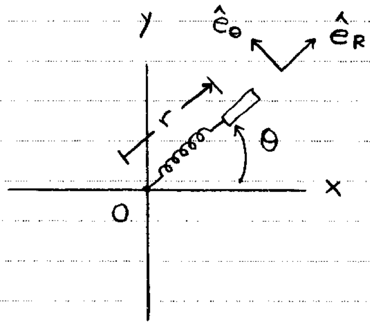
$-kr + kr_o = -mr\omega_o^2$

$r(k - m\omega_o^2) = kr_o$

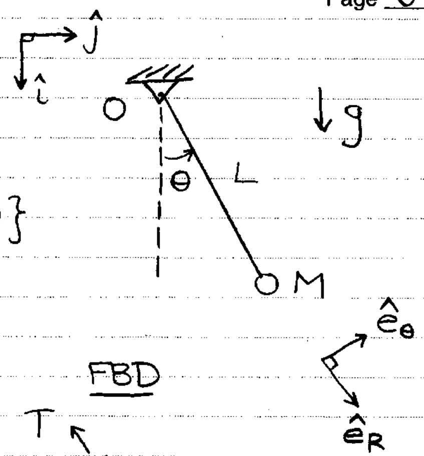
$\therefore \boxed{r = \frac{kr_o}{k - m\omega_o^2}}$

FBD





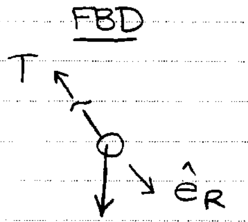
a) LMB : $\sum \underset{\sim}{F} = m\underset{\sim}{a}$



$$\{-T\hat{e}_R + mg\,\hat{\imath} = m(-L\dot{\theta}^2\hat{e}_R + L\ddot{\theta}\hat{e}_\theta)\}$$

$\{\}\cdot\hat{e}_\theta$

$$\Rightarrow \; mg(\hat{\imath}\cdot\hat{e}_\theta) = mL\ddot{\theta}$$



where

$$\hat{\imath}\cdot\hat{e}_\theta = \hat{\imath}\cdot(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{j}) = -\sin\theta$$

$$\therefore \; -mg\sin\theta = mL\ddot{\theta}$$

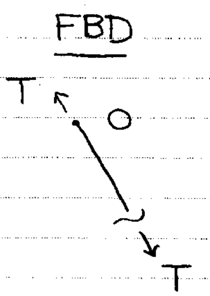
$$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

b) LMB $\cdot\,\hat{e}_R \Rightarrow -T + mg(\hat{\imath}\cdot\hat{e}_R) = -mL\dot{\theta}^2$

where $\hat{\imath}\cdot\hat{e}_R = \hat{\imath}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{j}) = \cos\theta$

$$\Rightarrow \boxed{T = mg\cos\theta + mL\dot{\theta}^2}$$

c) The force at the hinge support is simply $\underset{\sim}{F}_o = -T\hat{e}_R$ because the string is massless.



$$\underset{\sim}{F}_o = -T\hat{e}_R = -T(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{j})$$

$$\Rightarrow \boxed{\begin{aligned} \underset{\sim}{F}_o\cdot\hat{\imath} &= -T\cos\theta \\ \underset{\sim}{F}_o\cdot\hat{j} &= -T\sin\theta \end{aligned}}$$ where T is given above

d) Let $\alpha = \dot{\theta} \Rightarrow \dot{\alpha} = \ddot{\theta} = -\frac{g}{L}\sin\theta$

$$\Rightarrow \boxed{\begin{aligned} \dot{\theta} &= \alpha \\ \dot{\alpha} &= -\frac{g}{L}\sin\theta \end{aligned}}$$

e) see next page

(continued)

f) Max. tension = 29.9999... N

g) T = 2.3452 s $\leftarrow$ get by fine tuning tspan, interpolation, or event detection

```
global g L M

g=10; L=1; M=1;
tspan=[0 5];
theta0=pi/2; thetadot0=0;
z0=[theta0 thetadot0];
options=odeset('AbsTol',1e-12,'RelTol',1e-12);

[t,z]=ode45('deriv43',tspan,z0,options);

theta=z(:,1); thetadot=z(:,2);

tension=M*g*cos(theta)+M*L*thetadot.^2;
max(tension)

plot(t,theta)
hold on
plot(t,thetadot,'--')
plot(t,tension,':')
```

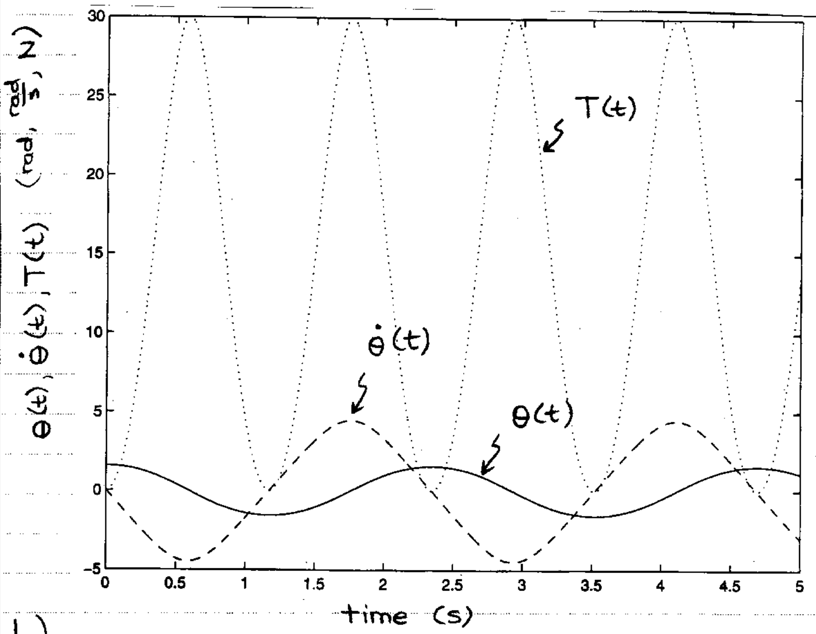
```
function zdot=deriv43(t,z)

global g L M

theta=z(1); alpha=z(2);

thetadot=alpha;
alphadot=(-g/L)*sin(theta);
zdot=[thetadot alphadot]';
```

Plots for 7.43



h)
Note: ① Maximum tension is independent of string length, ② The max. tension can be made closer to 30 N by decreasing the integration tolerance, ③ Using $\sin\theta \approx \theta$ get $T = 1.9869$ s (cf. $T = 2.3452$ s above).

$T = \frac{2\pi}{\lambda} = 2\pi\sqrt{\frac{L}{g}}$

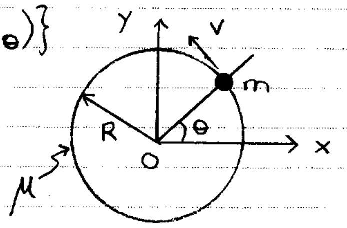
7.44

LMB: $\Sigma \underline{F} = m\underline{a}$

$\{-N\hat{e}_R - \mu N\hat{e}_\theta = m(-\frac{v^2}{R}\hat{e}_R + \dot{v}\hat{e}_\theta)\}$

$\{\}\cdot\hat{e}_R \Rightarrow \quad N = m\frac{v^2}{R} \quad (1)$

$\{\}\cdot\hat{e}_\theta \Rightarrow \quad -\mu N = m\dot{v} \quad (2)$



(1) into (2): $\boxed{\dot{v} = \frac{-\mu v^2}{R}}$

Since $v = R\dot{\theta}$,

$\frac{dv}{dt} = -\mu v \frac{d\theta}{dt}$

$\Rightarrow \frac{dv}{d\theta} = -\mu v$

$\therefore \boxed{v(\theta) = v_0 e^{-\mu\theta}}$ used $v(0) = v_0$

FBD