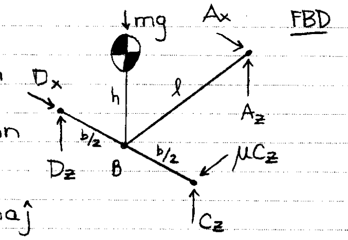
HW #6 - Solutions

6.74 A branch gets caught in the right rear wheel of a tricycle causing the wheel to skid in the $\hat{j}$-direction. The wheels at A & D roll without slip and are massless. What's the acceleration of the tricycle?

- A & D roll w/o slip in $\hat{j}$-direction
- wheel at C skids in $\hat{j}$-direction

FBD



LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = ma\hat{j}$

LMB $\cdot \hat{j} \Rightarrow -\mu C_z = ma$ (1)

$AMB_{/B} \cdot \hat{j}: \{\Sigma \underset{\sim}{M}_{/B} = \dot{\underset{\sim}{H}}_{/B}\}\cdot\hat{j}$ — moments about axis through B // to $\hat{j}$-direction

$\Rightarrow -C_z(\frac{b}{2}) + D_z(\frac{b}{2}) = 0$

$\therefore C_z = D_z$ (2)

$AMB_{/A} \cdot \hat{i}: \{\Sigma \underset{\sim}{M}_{/A} = \dot{\underset{\sim}{H}}_{/A}\}\cdot\hat{i}$ — moments about axis through A // to $\hat{i}$-direction

$\Rightarrow -C_z(\ell) - D_z(\ell) + mg(\ell) = -mah$

$mg - (C_z + D_z) = -ma\frac{h}{\ell}$ (3)

(2) into (3): $mg - 2C_z = -ma\frac{h}{\ell}$ (4)

(1) into (4): $\cancel{m}g + \frac{2\cancel{m}a}{\mu} = -\cancel{m}a\frac{h}{\ell}$

$a\left(\frac{2}{\mu} + \frac{h}{\ell}\right) = -g$

$\Rightarrow a = \dfrac{-g}{\frac{2}{\mu}+\frac{h}{\ell}} = \dfrac{-\frac{\mu g}{2}}{1+\frac{\mu h}{2\ell}}$

$$\boxed{\therefore \underset{\sim}{a} = -\frac{\mu g}{2}\left(\frac{1}{1+\frac{\mu h}{2\ell}}\right)\hat{j}}$$

Checks: ① When $\mu = 0$, $\underset{\sim}{a} = \underset{\sim}{0}$ (no forces in $\hat{j}$-direction $\Rightarrow$ constant $\underset{\sim}{v}$)

② When $h = 0$, $\underset{\sim}{a} = -\frac{\mu g}{2}\hat{j}$ (Half of the rider's weight is supported at wheel C.)

position: $\underset{\sim}{r}(t) = R\cos(\dot{\theta}t)\hat{i} + R\sin(\dot{\theta}t)\hat{j}$

where $\dot{\theta} = $ constant

a) $\underset{\sim}{v} = \dot{\underset{\sim}{r}} = R\dot{\theta}(-\sin(\dot{\theta}t)\hat{i} + \cos(\dot{\theta}t)\hat{j})$

$\therefore \underset{\sim}{v} \neq \underset{\sim}{0}$ since the velocity depends on time

b) $\underset{\sim}{v} \neq$ constant (see a))

c) $|\underset{\sim}{v}| = R\dot{\theta} = $ constant since $\dot{\theta} = $ const.

d) $\underset{\sim}{a} = \dot{\underset{\sim}{v}} = -R\dot{\theta}^2(\cos(\dot{\theta}t)\hat{i} + \sin(\dot{\theta}t)\hat{j})$

$\therefore \underset{\sim}{a} \neq \underset{\sim}{0}$ since the acceleration depends on time

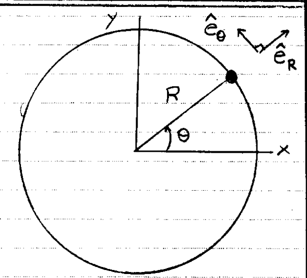
e) $\underset{\sim}{a} \neq$ constant (see d))

f) $|\underset{\sim}{a}| = R\dot{\theta}^2 = $ constant since $\dot{\theta} = $ const.

g) $\underset{\sim}{v}\cdot\underset{\sim}{a} = -R^2\dot{\theta}^3(-\sin(\dot{\theta}t)\cos(\dot{\theta}t) + \cos(\dot{\theta}t)\sin(\dot{\theta}t)) = \underline{\underline{0}}$

$\therefore \underline{\underline{\underset{\sim}{v} \perp \underset{\sim}{a}}}$

7.16



$\theta = \theta_0 \cos\lambda t$ (given)

$\Rightarrow \dot{\theta} = -\theta_0\lambda\sin\lambda t$

$\ddot{\theta} = -\theta_0\lambda^2\cos\lambda t$

$\underset{\sim}{a} = -R\dot{\theta}^2\hat{e}_R + R\ddot{\theta}\hat{e}_\theta$

where $\dot{\theta}^2 = \theta_0^2\lambda^2\sin^2\lambda t$
$= \lambda^2(\theta_0^2 - \theta_0^2\cos^2\lambda t)$
$= \lambda^2(\theta_0^2 - \theta^2)$

$\ddot{\theta} = -\lambda^2\theta$

get $|\underset{\sim}{a}|$ in terms of θ

$\Rightarrow |\underset{\sim}{a}|^2 = R^2\dot{\theta}^4 + R^2\ddot{\theta}^2$

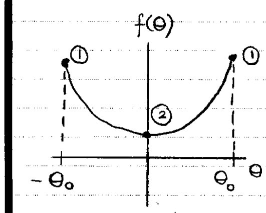
$|\underset{\sim}{a}|^2 = R^2[\lambda^4(\theta_0^2-\theta^2)^2 + \lambda^4\theta^2]$

$|\underset{\sim}{a}|^2(\theta) = R^2\lambda^4\underbrace{[(\theta_0^2-\theta^2)^2+\theta^2]}_{f(\theta)}$

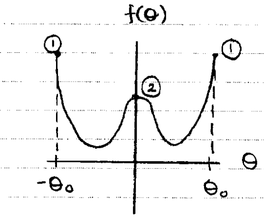
(continued)

Maximum acceleration occurs at the maximum value of

$$f(\theta) = (\theta_0^2-\theta^2)^2 + \theta^2 = \theta^4 + \theta^2(1-2\theta_0^2) + \theta_0^4$$

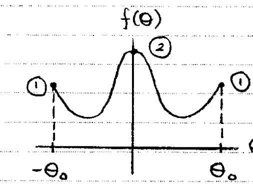


$\theta_0 < \frac{1}{2}$



$\frac{1}{2} < \theta_0 < 1$

① $f(\theta = |\theta_0|) = \theta_0^2$
② $f(\theta = 0) = \theta_0^4$



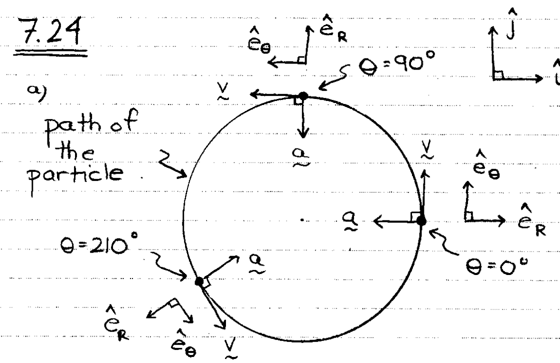
$\theta_0 > 1$

$\therefore$ Max value of $|\underset{\sim}{a}|$ will be at $\theta = 0$ only when $\underline{\theta_0 \geq 1}$.

$\Rightarrow \boxed{|\underset{\sim}{a}| = R\lambda^2\theta_0^2}$

7.24

a)



b) $\boxed{\dot{\theta} = \frac{2\pi}{\tau} = \text{constant}}$

@ $\underline{\theta = 0°}$: $\hat{e}_R = \hat{i}$, $\hat{e}_\theta = \hat{j}$
$\underset{\sim}{v} = R\dot{\theta}\hat{j}$, $\underset{\sim}{a} = -R\dot{\theta}^2\hat{i}$

@ $\underline{\theta = 90°}$: $\hat{e}_R = \hat{j}$, $\hat{e}_\theta = -\hat{i}$
$\underset{\sim}{v} = -R\dot{\theta}\hat{i}$, $\underset{\sim}{a} = -R\dot{\theta}^2\hat{j}$

(continued)

7.24 (cont'd)

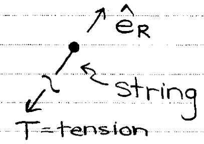
@ $\theta = 210°$: $\hat{e}_R = -\frac{\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}$

$\hat{e}_\theta = \frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}$

$\underset{\sim}{v} = R\dot{\theta}\left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$

$\underset{\sim}{a} = R\dot{\theta}^2\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$

c) LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$ — FBD



$\{-T\hat{e}_R = m(-R\dot{\theta}^2\hat{e}_R)\}$

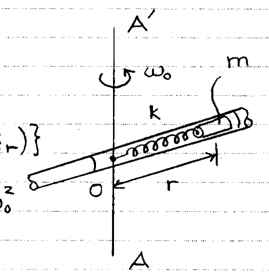
$\{\}\cdot\hat{e}_R \Rightarrow \boxed{T = mR\dot{\theta}^2}$

d) Since $\dot{\theta}$ = constant, there is no tangential acceleration. Therefore, the radial tension is all that's needed to keep the motion going.

e) $\underset{\sim}{T} = -T\hat{e}_R = T\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$ @ $\theta = 210°$

$\therefore \boxed{F_x = \frac{\sqrt{3}}{2}T = \frac{\sqrt{3}}{2}mR\dot{\theta}^2}$

7.29



LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$\{-k(r-r_0)\hat{e}_R = m(-r\omega_0^2\hat{e}_r)\}$

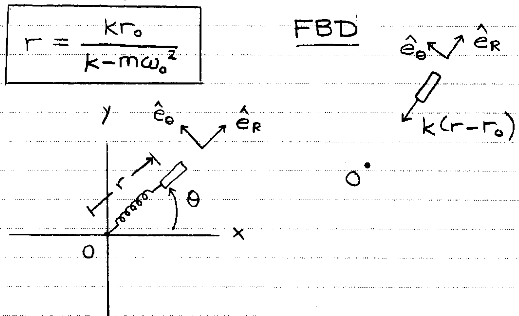
$\{\}\cdot\hat{e}_R \Rightarrow -k(r-r_0) = -mr\omega_0^2$

$-kr + kr_0 = -mr\omega_0^2$

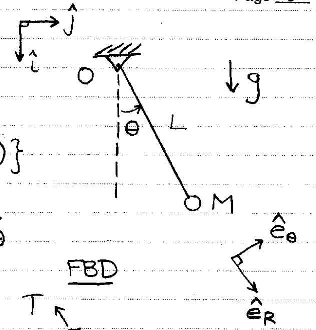
$r(k - m\omega_0^2) = kr_0$

$\therefore \boxed{r = \frac{kr_0}{k - m\omega_0^2}}$

FBD



7.43



a) LMB : $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$\{-T\hat{e}_R + mg\,\hat{\imath}$
$= m(-L\dot{\theta}^2\hat{e}_R + L\ddot{\theta}\hat{e}_\theta)\}$

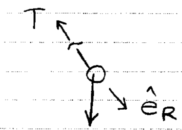
$\{\}\cdot\hat{e}_\theta$

$\Rightarrow mg(\hat{\imath}\cdot\hat{e}_\theta) = mL\ddot{\theta}$

where

$\hat{\imath}\cdot\hat{e}_\theta = \hat{\imath}\cdot(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})$
$= -\sin\theta$

FBD



$\therefore -mg\sin\theta = mL\ddot{\theta}$

$\Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$

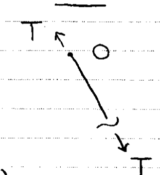
b) LMB $\cdot\,\hat{e}_R \Rightarrow -T + mg(\hat{\imath}\cdot\hat{e}_R) = -mL\dot{\theta}^2$

where $\hat{\imath}\cdot\hat{e}_R = \hat{\imath}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) = \cos\theta$

$\Rightarrow \boxed{T = mg\cos\theta + mL\dot{\theta}^2}$

c) The force at the hinge support is simply $\underset{\sim}{F}_o = -T\hat{e}_R$ because the string is massless.

FBD



$\underset{\sim}{F}_o = -T\hat{e}_R = -T(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$

$\Rightarrow \boxed{\begin{aligned}\underset{\sim}{F}_o\cdot\hat{\imath} &= -T\cos\theta \\ \underset{\sim}{F}_o\cdot\hat{\jmath} &= -T\sin\theta\end{aligned}}$ where T is given above

d) Let $\alpha = \dot{\theta} \Rightarrow \dot{\alpha} = \ddot{\theta} = -\frac{g}{L}\sin\theta$

$\Rightarrow \boxed{\begin{aligned}\dot{\theta} &= \alpha \\ \dot{\alpha} &= -\frac{g}{L}\sin\theta\end{aligned}}$

e) see next page

(continued)

f) Max. tension = 29.9999... N

g) T = 2.3452 s ← get by fine tuning tspan, interpolation, or event detection

7.43 (contd)


```
global g L M

g=10; L=1; M=1;
tspan=[0 5];
theta0=pi/2; thetadot0=0;
z0=[theta0 thetadot0];
options=odeset('AbsTol',1e-12,'RelTol',1e-12);

[t,z]=ode45('deriv43',tspan,z0,options);

theta=z(:,1); thetadot=z(:,2);

tension=M*g*cos(theta)+M*L*thetadot.^2;
max(tension)

plot(t,theta)
hold on
plot(t,thetadot,'--')
plot(t,tension,':')
```

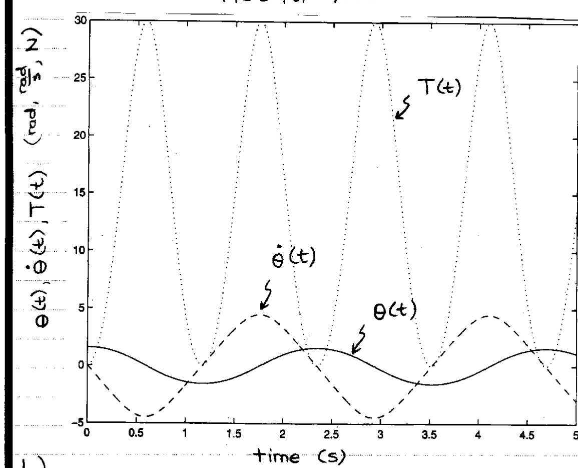
```
function zdot=deriv43(t,z)

global g L M

theta=z(1); alpha=z(2);

thetadot=alpha;
alphadot=(-g/L)*sin(theta);
zdot=[thetadot alphadot]';
```

Plots for 7.43

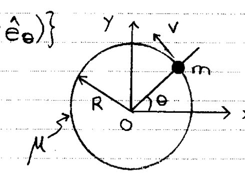


h)

Note: ① Maximum tension is independent of string length, ② The max. tension can be made closer to 30 N by decreasing the integration tolerance, ③ Using $\sin\theta \approx \theta$ get T = 1.9869 s (cf. T = 2.3452 s above).

$T = \frac{2\pi}{\lambda} = 2\pi\sqrt{\frac{L}{g}}$

7.44



LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$

$\{-N\hat{e}_R - \mu N\hat{e}_\theta = m\left(-\frac{v^2}{R}\hat{e}_R + \dot{v}\hat{e}_\theta\right)\}$

$\{\}\cdot\hat{e}_R \Rightarrow \quad N = m\frac{v^2}{R} \quad (1)$

$\{\}\cdot\hat{e}_\theta \Rightarrow -\mu N = m\dot{v} \quad (2)$

(1) into (2): $\boxed{\dot{v} = -\frac{\mu v^2}{R}}$

Since $v = R\dot{\theta}$,

$\frac{dv}{dt} = -\mu v\frac{d\theta}{dt}$

$\Rightarrow \frac{dv}{d\theta} = -\mu v$

$\therefore \boxed{v(\theta) = v_0 e^{-\mu\theta}}$ ← used $v(0) = v_0$

FBD