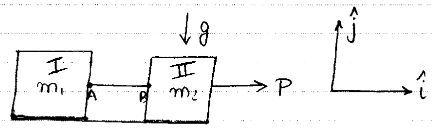
T&AM 203 HW #5

DUE 2/29/00 - PRITAM GANGULY

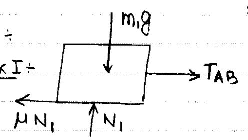
(6.2)



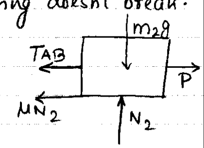
To find the max. value of P so that string doesn't break.

FBDs :

BLOCK I :



BLOCK II :



LMB : BLOCK I : $T_{AB}\hat{i} - \mu N_1\hat{i} + N_1\hat{j} - m_1 g\hat{j} = m_1 a_1 \hat{i}$ — (A)

Eqn (A) $\cdot \hat{i} \Rightarrow$ $T_{AB} - \mu N_1 = m_1 a_1$ — (1)

Eqn (A) $\cdot \hat{j} \Rightarrow$ $N_1 = mg$ — (2)

(2) in (1) $\Rightarrow$ $\boxed{T_{AB} - \mu m_1 g = m_1 a_1}$ — (3)

BLOCK II : $P\hat{i} - T_{AB}\hat{i} - \mu N_2\hat{i} + N_2\hat{j} - m_2 g\hat{j} = m_2 a_2 \hat{i}$ — (B)

Eqn (B) $\cdot \hat{i} \Rightarrow$ $P - T_{AB} - \mu N_2 = m_2 a_2$ - (4)

Eqn (B) $\cdot \hat{j} \Rightarrow$ $N_2 = m_2 g$ - (5)

(5) in (4) $\Rightarrow$ $\boxed{P - T_{AB} - \mu m_2 g = m_2 a_2}$ (6)

Looking @ the constraints as length of string AB doesn't change

$a_A = a_B \Rightarrow a_1 = a_2 = a$ (say) - (7)

Also given $m_1 = m_2 = m$ - (8)

Using these facts ((7) & (8)) and the equations (3) & (6) (subtracting (3) from (6)) we get

$P - 2T_{AB} = 0 \Rightarrow \boxed{T_{AB} = P/2}$

Since the critical tension a string can hold before snapping is given as "T_{cr}"

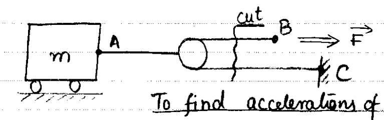
$\therefore$ Max. tension possible in the string $= T_{cr} = \frac{P_{max}}{2}$

$\therefore \boxed{P_{max} = 2T_{cr}}$

So the max force applied should be just less than $2T_{cr}$ so that the string doesn't snap.

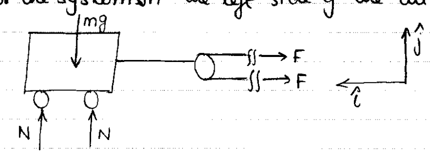
6.13(b)

(6.13(b))



To find acceleration of A & B

FBD for the system on the left side of the cut



Assumptions : (a) No friction (b) inextensible chord (massless) (c) pulley of negligible mass.

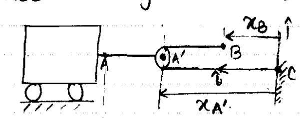
from these assumptions (a & b), tension in the chord throughout is same & equal to F.

LMB in the $\hat{i}$ direction gives

$-2F = m a_A$

$\Rightarrow \boxed{a_A = -\frac{2F}{m}}$ (note that +ve $\hat{i}$ axes is directed to the left).

To get a_B, look at the constraints. choose C as origin & same set of axes.



let pulley radius be r.

The length of string C through over the pulley to B is a constant

So $x_{A'} + \pi r + (x_{A'} - x_B) = \ell$ (length of string) — (*)

Differentiating twice wrt. time

$2\ddot{x}_{A'} = \ddot{x}_B$ — (1)

Now, A moves the same distance as does A' ($\because AA' =$ constant)

$\therefore a_A = a_{A'}$

But from (1) $a_{A'} = \frac{a_B}{2}$

$\therefore a_A = \frac{a_B}{2} \Rightarrow \boxed{a_B = 2a_A}$

$\therefore \boxed{a_B = -\frac{4F}{m}}$

(6.14 b) To solve 6.13(b) using power balance lets use the same set of axes as above

Power balance equation : $P = \dot{E}_K$

The only external force to the whole system which does work is the force F

contd ⟶

6.14(b) contd.

$\therefore F(-\hat{i}) \cdot v_B(\hat{i}) = \frac{d}{dt}\left(\frac{1}{2} m v_A^2\right)$

(note again that +ve $\hat{i}$ is to the left).

$\Rightarrow -F v_B = \frac{1}{2} m \cdot 2 v_A \cdot \dot{v}_A = m v_A a_A$

$\Rightarrow \boxed{a_A = \frac{-F}{m} \cdot \frac{v_B}{v_A}}$ — (1)

from the constraint equations in 6.13(b) Eqn (*)

$x_{A'} + \pi r + x_{A'} - x_B = \ell$

Differentiating once,

$2\dot{x}_{A'} = \dot{x}_B$

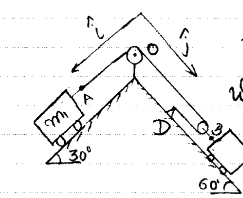
$\Rightarrow 2\dot{x}_A = \dot{x}_B$

$\Rightarrow 2v_A = v_B \Rightarrow \frac{v_B}{v_A} = 2$ — (2)

Using (2) in (1) $\boxed{a_A = -\frac{2F}{m}}$ Ans

again by constraint eqn $\boxed{a_B = 2a_A = -\frac{4F}{m}}$

6.18(b).



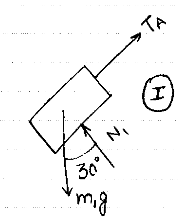
find a_A, a_B

We ignore friction here

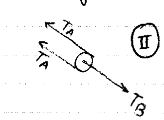
LMBs : (We will just write those eqns which have consequence).

FBDs :

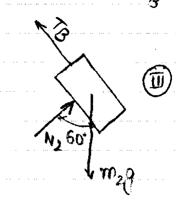
for m_1 :



pulley.



m_2 :



I : $(T_A - m_1 g \sin 30)\hat{i} + m_1 g \cos 30°\hat{j} - N_1\hat{j} = m_1 a_1 \hat{i}$

$\{I\} \cdot \hat{i} \Rightarrow T_A - \frac{m_1 g}{2} = m_1 a_1$ — (1)

II : $\left\{-2T_A\hat{j} + T_B\hat{j} = \underset{=0}{m_{pulley}} a\right\} \cdot \hat{j}$

$\Rightarrow 2T_A = T_B$ — (2)

III : $(-T_B + m_2 g \sin 60°)\hat{j} + (m_2 g \cos 60° - N_2)\hat{i}$

$= m_2 a_2 \hat{j}$

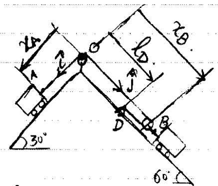
$\{III\} \cdot \hat{j} \Rightarrow -T_B + m_2 g \cdot \frac{\sqrt{3}}{2} = m_2 a_2$ — (3)

Adding $2 \times$ eqn (1) to (3) & using eqn (2) to eliminate T_B we get $\boxed{m_2 g \frac{\sqrt{3}}{2} - m_1 g = 2m_1 a_1 + m_2 a_2}$ — (4)

contd ⟹

6.18 (b) contd

Now use contraints to relate a_1 & a_2.
Choose O as the origin of the same set of coord. axes as used before.



Choose distances as shown.
Notice that l_D which is distance between O & D is a constant.
Let l be the total length of string (neglecting the part which goes over the pulley because anyway it gets killed while differentiating!).

$$x_A + x_B + x_B - l_D = l.$$

Diff. twice $\quad a_A = -2a_B \Big\} - ⑤$
$\Rightarrow a_1 = -2a_2$

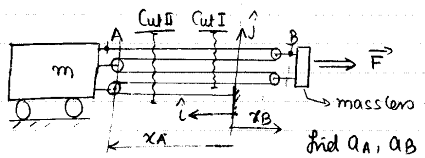
Using ⑤ in ④

$$m_2 g\frac{\sqrt{3}}{2} - m_1 g = (-4m_1 + m_2)\,a_2$$

$$\Rightarrow \boxed{a_2 = +\frac{g\,(\sqrt{3}m_2 - 2m_1)}{2\,(m_2 - 4m_1)}}$$

$$\boxed{a_1 = -2a_2 = -\frac{g\,(\sqrt{3}m_2 - 2m_1)}{(m_2 - 4m_1)}}$$

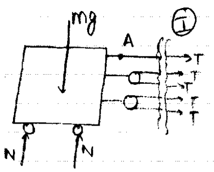
(6.18(c))



find a_A, a_B

Drawing FBDs for right part of Cut I & left part of Cut II
Let tension in the string starting from A be T.

Cut I -

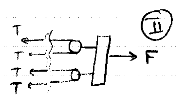


LMB :-
for I along $\hat{i}$ direction

$$\Big\{-5T\hat{i} = ma_A\hat{i}\Big\}\cdot\hat{i}$$

$\Rightarrow -5T = ma_A \Rightarrow \boxed{a_A = \frac{-5T}{m}}$

Cut II -



for II along $\hat{i}$; $\quad 4T\hat{i} = F\hat{i} = \underbrace{m}_{=0}a$

$\Rightarrow 4T = F$

$\therefore \boxed{a_A = -\frac{5F}{4m}}$

contd →

6.18(c) contd

For a_B, constraints

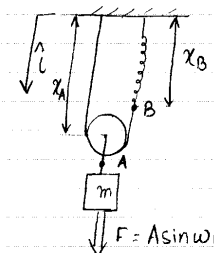
$$x_A + x_A + (x_B) + (-x_B) + (x_A) + x_A + (-x_B) + (-x_B) + (x_A) = l \text{ (length of string)}$$

$\Rightarrow 5x_A - 4x_B = l \qquad$ Diff. twice we get $5a_A = 4a_B$

$$\therefore \boxed{a_B = \frac{5}{4}a_A = \frac{-25F}{16m}}$$

(note again +ve $\hat{i}$ is to the left).

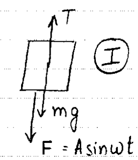
(6.27(a))

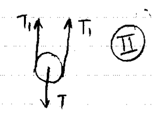


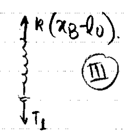
To write the differential equ of motion for the system.

Let l_0 be initial (relaxed) length of spring.

FBDs :







LMBs :

Ⅲ $\Rightarrow \Big\{T_1\hat{i} - k(x_B - l_0)\hat{i} = 0\Big\}\cdot\hat{i}$

$\Rightarrow T_1 = k(x_B - l_0)$.

Ⅱ $\Rightarrow T = 2T_1$

$\Rightarrow T = 2k(x_B - l_0)$.

Ⅲ $\Rightarrow \Big\{(mg + F)\hat{i} - T\hat{i} = ma_A\hat{i} = m\ddot{x}_A\hat{i}\Big\}\cdot\hat{i}$

$\Rightarrow mg + F - 2k(x_B - l_0) = m\ddot{x}_A$ — ①

To write x_B interms of x_A look @ constraints.

Let l be total length of string

$x_A + x_A - x_B = l - \pi r \qquad r \to$ radius of pulley

$x_B = 2x_A - l + \pi r$ — ②

② in ① $\Rightarrow$

$$mg + A\sin\omega t = m\ddot{x}_A + 2k(2x_A - l + \pi r - l_0)$$

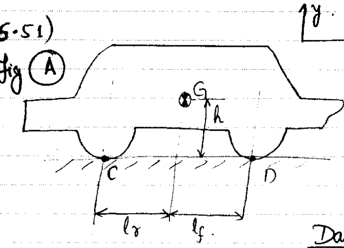
$$\Rightarrow \boxed{m\ddot{x}_A + 4kx_A = mg + A\sin\omega t + 2k(l + l_0 - \pi r)}.$$

(Observe that using a pulley system like this increases the natural frequence $\sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$ two times than a mass-spring system).

6.51

(6.51)

Fig Ⓐ

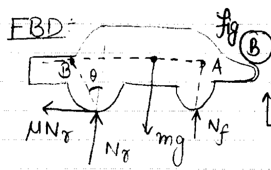


To know the deceleration, reaction forces for different cases of skidding.

Data : $l_r = l_f = \frac{\omega}{2}$
$\mu = 1$

(a) FBD :-

Fig Ⓑ

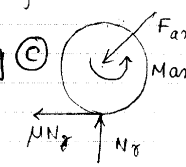


Only the rear wheel skidding (friction acts opposite to relative velocity) so when it skids while braking, the friction acts opposite to direction of motion

where $\theta = \tan^{-1}\frac{1}{\mu} = \tan^{-1} 1 = 45°$ (in our case)

FBD of the rear wheel

Fig Ⓒ



Note that in the FBD of rear wheel there is a reaction moment from the axle.

In fig Ⓑ points A & B are nice to write AMB about.

for eg. about A when we write AMB the only unknown is N_r coz the term $\vec{r}\times m\vec{a}_{cm}$ vanishes since $\vec{a}_{cm}$ passes through A

Similarly when we consider B, the only unknown is N_f in the AMB equation.

(b) LMB for (a).

$$-\mu N_r\hat{i} + N_r\hat{j} + N_f\hat{j} - mg\hat{j} = ma\hat{i} \text{ — Ⅰ}$$

Ⅰ$\cdot\hat{i} \Rightarrow -\mu N_r = ma$ — ①

Ⅰ$\cdot\hat{j} \Rightarrow N_r + N_f = mg$ — ②

(c) Take moments about A & writing AMB about A

$$\left(mg\frac{\omega}{2} - N_r\omega - \mu N_r h\right)\hat{k} = -\left(\frac{\omega}{2}\hat{i}\times ma\hat{i}\right) = 0$$

⟶ contd

(d) $\therefore\ mg\frac{\omega}{2} - N_r\omega - \mu N_r h = 0$

$$\Rightarrow \boxed{N_r = \frac{mg\omega}{2(\omega+\mu h)}} \quad — (3)$$

from (2), $N_f = mg - N_r = mg - \frac{mg\omega}{2(\omega+\mu h)}$

$$\Rightarrow \boxed{N_f = \frac{mg(\omega+2\mu h)}{2(\omega+\mu h)}} \quad — (4)$$

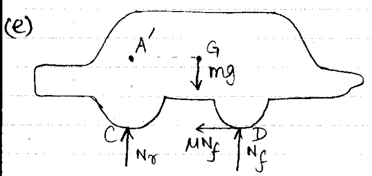
from (1) $\boxed{a = \frac{-\mu g\omega}{2(\omega+\mu h)}}$ — (5)

Aside ÷
looking @ the expressions we see that $\mu = 0$ (no friction) the $a = 0$ & hence no deceleration

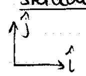
we see from (5) that $|a| \leq \frac{\mu g}{2}$

the equality comes (i.e max deceleration is reached if $\omega >> h$) or $h = 0$

(e)



front wheel skidding



Writing AMB about A'

$$\left(-\mu N_f h - mg\frac{\omega}{2} + N_f\cdot\omega\right)\hat{k} = \frac{\omega}{2}\hat{i} \times ma'\hat{i} = 0$$

$$\Rightarrow \boxed{N_f = \frac{mg\omega}{2(\omega-\mu h)}} \quad — (6)$$

LMB ÷ $\left\{-\mu N_f\hat{i} + (N_f + N_r - mg)\hat{j} = ma'\hat{i}\right\}$ — (II)

(II)$\cdot\hat{i} \Rightarrow\ -\frac{\mu N_f}{m} = a \Rightarrow \boxed{a' = \frac{-\mu g\omega}{2(\omega-\mu h)}}$ — (7)

(II)$\cdot\hat{j} \Rightarrow\ N_f + N_r = mg$

$$\Rightarrow \boxed{N_r = mg - N_f = mg - \frac{mg\omega}{2(\omega-\mu h)} = \frac{mg(\omega-2\mu h)}{2(\omega-\mu h)}} \quad — (8)$$

→ contd.

(e) contd. Look at equations (5) & (7)
denom. of (5) > denominator of (7)

$\therefore\ |a'| \geq |a|$ so the deceleration in the case of front wheel skidding is larger & so the car comes to halt faster in this case.

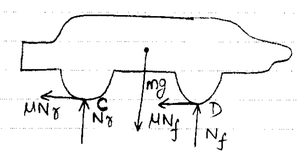
If $h = 0$ then we can choose A' again but now it coincides with point ~~G~~ C.
& so there is no moment due to the frictional force about A',
but we can derive this case from the equations derived in terms of h.
looking @ (6) & (7)

if h=0
$\Rightarrow \boxed{N_f = N_r = \frac{mg}{2}}$ $\boxed{a' = -\frac{\mu g}{2}}$ → front wheel skidding

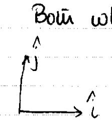
from (5) we have $\boxed{a = -\frac{\mu g}{2}}$ → rear wheel skidding

Since deceleration is the same, so both cases car would take the same time to come to a halt.

(f)



Both wheels skidding



LMB ÷ $(-\mu N_r - \mu N_f)\hat{i} + (N_r + N_f - mg)\hat{j} = m\bar{a}\hat{i}$ — (III)

(III)$\cdot\hat{i} \Rightarrow\ -\mu(N_r + N_f) = m\bar{a}$ - (9)

(III)$\cdot\hat{j} \Rightarrow\ (N_r + N_f) = mg$ — (10)

Using (10) in (9) $\boxed{\bar{a} = -\frac{\mu \cancel{m} g}{\cancel{m}} = -\mu g}$ — (11)

AMB about C ÷ $\left\{\left(N_f\cdot\omega - mg\frac{\omega}{2}\right)\hat{k} = -m\bar{a}h\hat{k}\right\}\cdot\hat{k}$

$$\Rightarrow \boxed{N_f = \frac{mg\left(\frac{\omega}{2} + \mu h\right)}{\omega}} \quad — (12)$$

$$\boxed{N_r = mg - N_f = \frac{mg\left(\frac{\omega}{2} - \mu h\right)}{\omega}} \quad — (13)$$

→ contd

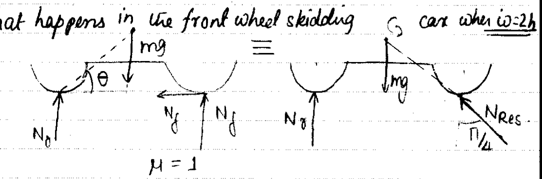
(g) from (5) and (7)

we had $a = \frac{-\mu g\omega}{2(\omega+\mu h)}$ $a' = \frac{-\mu g\omega}{2(\omega-\mu h)}$.

$$a + a' \neq \bar{a}$$

We saw that in front wheel & rear wheel skidding when AMB was written about A' & A respectively though the situation looked identical but moments added up to be different & this difference shows up in the denominator of the normal forces, hence the dependence is not linear
& hence principle of superposition won't do.

h) What happens in the front wheel skidding car when $\omega = 2h$



$\therefore\ h = \frac{\omega}{2}$

$\therefore\ \theta = \tan^{-1}\frac{h}{\omega/2} = \frac{\pi}{4}$

So we see that resultant reaction force at the front wheel passes through CM

So take G as the point for AMB

$N_r = 0$. (this can be got by substituting $\omega = 2h$ & $\mu = 1$ in eqn (8)).

So Reaction at rear wheel vanishes.

(i) If $\omega < 2h$ $(\mu = 1)$
then from eqn (8)

$N_r < 0$ which isn't a possible situation since ground can't suck the tyre in

This comes out as a result coz once normal reaction becomes zero, there is no more skidding but the car is toppling over about the front wheel.