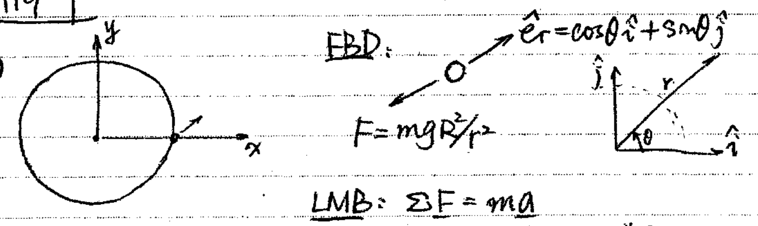
#1 T&AM 203 - DYNAMICS (HW#4 - 2/22/(

5.119

a)



LMB: $\Sigma \underline{F} = m\underline{a}$

$$mgR^2/r^2(-\hat{e}_r) = m\ddot{x}\hat{i} + m\ddot{y}\hat{j}$$

LMB $\cdot\,\hat{i}$: $m\ddot{x} + mgR^2\frac{x}{(x^2+y^2)^{3/2}} = 0$ $\qquad r = \sqrt{x^2+y^2}$

$\hat{e}_r\cdot\hat{i} = \cos\theta = \frac{x}{r}$

LMB $\cdot\,\hat{j}$: $m\ddot{y} + mgR^2\frac{y}{(x^2+y^2)^{3/2}} = 0$ $\qquad \hat{e}_r\cdot\hat{j} = \sin\theta = \frac{y}{r}$

Let $z_1 = x$, $z_2 = \dot{x}$, $z_3 = y$, $z_4 = \dot{y}$

$$\Rightarrow \left.\boxed{\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -gR^2\frac{z_1}{(z_1^2+z_3^2)^{3/2}} \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= -gR^2\frac{z_3}{(z_1^2+z_3^2)^{3/2}} \end{aligned}}\right\}$$

1st order system written for numerical solution

Initial Conditions:

$t=0$, $x=R$, $y=0$, $\dot{x}=\frac{v_0}{\sqrt{2}}$, $\dot{y}=\frac{v_0}{\sqrt{2}}$

$\Rightarrow$ $z_1=R$, $z_2=\frac{v_0}{\sqrt{2}}$, $z_3=0$, $z_4=\frac{v_0}{\sqrt{2}}$

b)

```
%Matlab solutution to problem 5.119

global g R v0

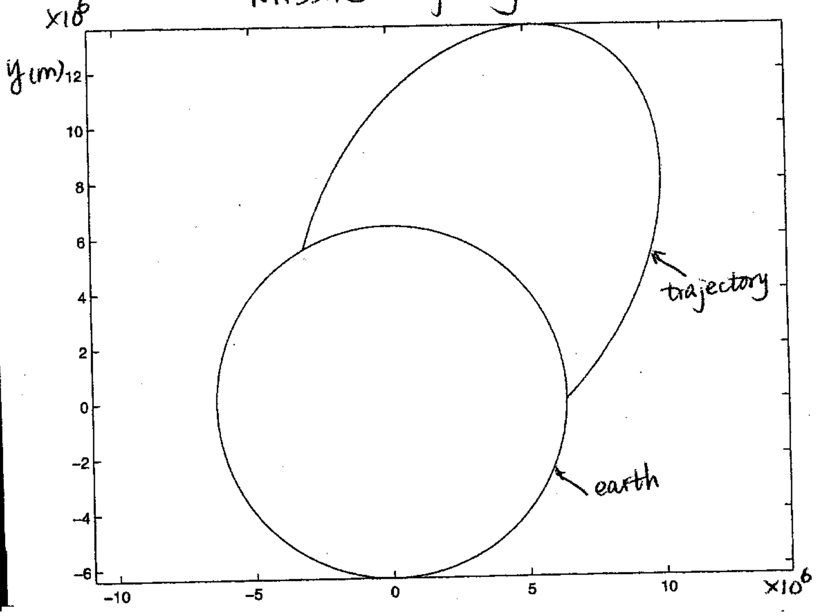
g=10; R=6400000; v0=9000;
tspan=[0 6670];
z0=[R;v0/sqrt(2);0;v0/sqrt(2)];
[t,z]=ode45('deriv1',tspan,z0);
x=z(:,1);y=z(:,3);

plot(x,y)
```

```
function zdot=deriv1(t,z);

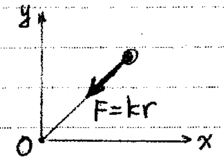
global g R v0
zdot=[z(2);-g*R*R*z(1)/(z(1)*z(1)+z(3)*z(3))/sqrt(z(1)*z(1)+z(3)*z(3));
      z(4);-g*R*R*z(3)/(z(1)*z(1)+z(3)*z(3))/sqrt(z(1)*z(1)+z(3)*z(3))];
```

Missile Trajectory in Prob. 5.119



$\boxed{5.120}$

a) FBD:



$\underline{r} = x\hat{i} + y\hat{j}$

LMB: $\Sigma \underline{F} = m\underline{a}$ $\quad -k\underline{r} = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$

LMB$\cdot\hat{i}$: $m\ddot{x} + kx = 0$ ①

LMB$\cdot\hat{j}$: $m\ddot{y} + ky = 0$ ②

the general soln of ①:

$$x = A\sin(\sqrt{\tfrac{k}{m}}t) + B\cos(\sqrt{\tfrac{k}{m}}t)$$

the general soln of ②:

$$y = C\sin(\sqrt{\tfrac{k}{m}}t) + D\cos(\sqrt{\tfrac{k}{m}}t)$$

it's possible that the trajectory is a circle.

i.e. we pick $x = A\sin(\sqrt{\tfrac{k}{m}}t)$, $y = A\cos(\sqrt{\tfrac{k}{m}}t)$

thus $x^2 + y^2 = A^2$

hence, $r_0 = A$.

$v_0 = \sqrt{\dot{x}^2 + \dot{y}^2} = A\sqrt{\tfrac{k}{m}}$

$\therefore$ the relation between r_0 & v is:

$$\boxed{v_0 = r_0\sqrt{\tfrac{k}{m}}}$$

b) since there is no external moment acting on the system, the angular momentum is conserved. $(\Sigma \underline{M}_{/o} = \underline{r} \times \underline{F} = \underline{r} \times (-k\underline{r}) = 0)$

thus $H_0 = mv_1 b = mv_2 a$. $(\underline{H} = \underline{r} \times m\underline{v} = mvr\hat{k})$

$\Rightarrow v_2 = \frac{b}{a}v_1$

the original elliptical trajectory can be seen as:

$x = a\sin(\omega t + \phi)$ $\qquad \dot{x} = a\omega\cos(\omega t + \phi)$

$y = b\cos(\omega t + \phi)$ $\qquad \dot{y} = b\omega\sin(\omega t + \phi)$

Now change to the circle orbit:

$x = A\sin(\omega t + \Phi)$ $\qquad \dot{x} = A\omega\cos(\omega t + \Phi)$

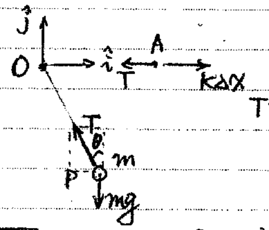
$y = A\cos(\omega t + \Phi)$ $\qquad \dot{y} = A\omega\sin(\omega t + \Phi)$

at point ②: speed $v_2 = b\omega \xrightarrow{\text{increases}} v_2' = a\omega$.

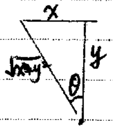
$\therefore$ $\boxed{\Delta v = v_2' - v_2 = (a-b)\omega = (a-b)\frac{v_1}{a} = \frac{v_1}{5} \text{ in vertical}}$

#5.111

a). FBD



$T = k\Delta x$



$\sin\theta = \frac{x}{\sqrt{x^2+y^2}}$

$\cos\theta = \frac{-y}{\sqrt{x^2+y^2}}$ (since y is negative)

$$\Delta x = (\underbrace{\sqrt{x^2+y^2}}_{|OP|} + \underbrace{10}_{|OB|}) - (\underbrace{8}_{l_r} + \underbrace{2}_{l_0}) = \sqrt{x^2+y^2}$$

$\therefore$ LMB: $\sum \underline{F} = m\underline{a}$

LMB $\cdot \hat{i}$: $-k\sqrt{x^2+y^2}\cdot\left(\frac{x}{\sqrt{x^2+y^2}}\right) = m\ddot{x} \Rightarrow m\ddot{x} + kx = 0$

LMB $\cdot \hat{j}$: $k\sqrt{x^2+y^2}\cdot\left(\frac{-y}{\sqrt{x^2+y^2}}\right) = m\ddot{y} + mg \Rightarrow m\ddot{y} + ky + mg = 0$

$\therefore x = A\sin(\sqrt{\frac{k}{m}}t) + B\cos(\sqrt{\frac{k}{m}}t)$

From the initial condition.

$x(0) = B = x_0$, $\dot{x}(0) = A\sqrt{\frac{k}{m}} = \dot{x}_0$

$\therefore x = \frac{\dot{x}_0}{\sqrt{\frac{k}{m}}}\sin(\sqrt{\frac{k}{m}}t) + x_0\cos(\sqrt{\frac{k}{m}}t) = \frac{\dot{x}_0}{\sqrt{2}}\sin\sqrt{2}t + x_0\cos\sqrt{2}t$

Similarly, $y = C\sin(\sqrt{\frac{k}{m}}t) + D\cos(\sqrt{\frac{k}{m}}t) - \frac{m}{k}g$

From the initial condition

$y(0) = D - \frac{m}{k}g$, $\dot{y}(0) = C\sqrt{\frac{k}{m}} = \dot{y}_0$

$\therefore y = \frac{\dot{y}_0}{\sqrt{\frac{k}{m}}}\sin(\sqrt{\frac{k}{m}}t) + (\frac{m}{k}g + y_0)\cos(\sqrt{\frac{k}{m}}t) - \frac{m}{k}g$

$= \frac{\dot{y}_0}{\sqrt{2}}\sin(\sqrt{2}t) + (5+y_0)\cos(\sqrt{2}t) - 5$

b)

```
%Matlab solutution to problem 5.111

global m g k

g=10; m=100; k=200;
tspan=[0 pi/sqrt(2)];
z0=[1;0;-5;0];
[t,z]=ode45('deriv2',tspan,z0);
x=z(:,1);y=z(:,3);


x=-1 , y=-5
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$x = -1$ (m), $y = -5$ (m) @ $t = \pi/\sqrt{2}$ (s)

```
function zdot=deriv2(t,z);

global g m k
zdot=[z(2);-k*z(1)/m;
      z(4);-g-k*z(3)/m];
```

c). when $\underline{r}_0 = 1m\hat{i} - 5m\hat{j}$, $\underline{v}_0 = 0$

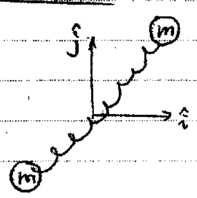
then $x = x_0\cos(\sqrt{\frac{k}{m}}t) = 1\cdot\cos(\sqrt{2}t)$

@ $t = \pi/\sqrt{2}$ s. $\boxed{x = 1\cdot\cos\pi = -1\ m}$

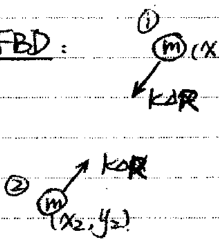
$\boxed{y = (5+y_0)\cos(\sqrt{2}t) - 5 = -5\ m}$ @ $t = \pi/\sqrt{2}$ s

pies.

#5.124



FBD:



a) Since there is no external forces acting on the whole system, the total linear momentum is conserved.

b) the mass center can't accelerate, since no external force. Otherwise a) will also violates.

c) see above.

d) LMB: $\sum \underline{F} = m\underline{a}$

since the masscenter doesn't accelerate, we can setup an x-y frame there.

$\Delta x: \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - 2R$

①: $$-k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-2R\right)\frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = m\ddot{x}_1$$

$$-k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-2R\right)\frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = m\ddot{y}_1$$

②: $$k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-2R\right)\frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = m\ddot{x}_2$$

$$k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-2R\right)\frac{y_1-y_2}{\sqrt{(y_1-y_2)^2+(x_1-x_2)^2}} = m\ddot{y}_2$$

since the masscenter won't change

$\therefore x_1 = -x_2,\ y_1 = -y_2$ $\therefore$ ①&② are just in the opposite direction relative to the origin.

e) No friction the total energy is conserved $E_K + E_P = \text{const} = E_{K_0} + E_{P_0}$

$$\boxed{\begin{aligned} K_E &= \tfrac{1}{2}m(\dot{x}_1^2+\dot{y}_1^2)+\tfrac{1}{2}m(\dot{x}_2^2+\dot{y}_2^2) \\ K_P &= \tfrac{1}{2}k\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}-2R\right)^2 \end{aligned}}$$

* Note: We assume that the initial velocity of the system (CM) is zero.

pies.
r

f)

```
%Matlab solutution to problem 5.124

global m R k

R=5; m=2; k=10;
tspan=[0 10];
z0=[5;-1;5;2];
[t,z]=ode45('deriv3',tspan,z0);
x1=z(:,1);y1=z(:,3);
x2=-x1;y2=-y1;

plot(x1,y1);
hold on;
plot(x2,y2,'--');
Axis equal
```

```
function zdot=deriv2(t,z);

global R m k
zdot=[z(2);-2*k/m*z(1)*(sqrt(z(1)*z(1)+z(3)*z(3))-R)/sqrt(z(1)*z(1)+z(3)*z(3));
      z(4);-2*k/m*z(3)*(sqrt(z(1)*z(1)+z(3)*z(3))-R)/sqrt(z(1)*z(1)+z(3)*z(3))]
```

plot of mass1,2's motion (keep mass center fixed)