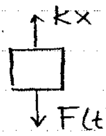
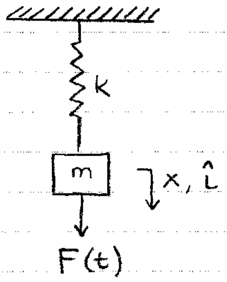
due 2/15/00 - David Cabrera

5.41

FBD





LMB: $\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$F(t)\hat{i} - kx\hat{i} = m\ddot{x}\hat{i}$

LMB $\cdot \hat{i} \Rightarrow \quad \ddot{x} + \frac{k}{m}x = \frac{1}{m}F(t) \quad (1)$

where $F(t) = F_o \sin pt$

and $F_o = 5\,N$ & $p = 2\pi$ rad/s

Particular soln.: $x_p(t) = A\sin pt + B\cos pt$

$\Rightarrow \dot{x}_p(t) = pA\cos pt - pB\sin pt$

$\quad \ddot{x}_p(t) = -p^2 A\sin pt - p^2 B\cos pt$

Plugging into (1):

$$-p^2 A\sin pt - p^2 B\cos pt + \frac{k}{m}A\sin pt + \frac{k}{m}B\cos pt = \frac{1}{m}F_o \sin pt$$

sin(pt) terms: $-p^2 A + \frac{k}{m}A = \frac{1}{m}F_o$

cos(pt) terms: $-p^2 B + \frac{k}{m}B = 0$

$$\Rightarrow B = 0, \quad A = \frac{F_o/m}{\frac{k}{m} - p^2}$$

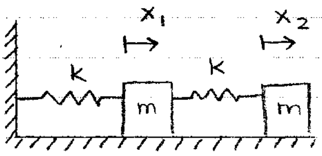
ignore the "homogeneous" solution

$$\therefore \boxed{x_p(t) = \frac{F_o/m}{k/m - p^2}\sin pt}$$

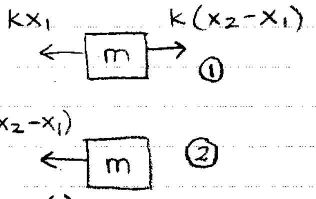
$$\text{Amplitude} = \left|\frac{F_o/m}{k/m - p^2}\right| = \left|\frac{5\,N/3\,kg}{\frac{10\,N/m}{3\,kg} - (2\pi \frac{rad}{s})^2}\right|$$

$$\boxed{\therefore \text{Amplitude} = 0.0461\ m}$$

5.52



FBDs



LMB①: $\sum \underset{\sim}{F} = m\underset{\sim}{a}_1$

LMB①$\cdot \hat{i} \Rightarrow -kx_1 + k(x_2 - x_1) = m\ddot{x}_1 \quad (1)$

LMB②: $\sum \underset{\sim}{F} = m\underset{\sim}{a}_2$

LMB②$\cdot \hat{i} \Rightarrow -k(x_2 - x_1) = m\ddot{x}_2 \quad (2)$

(continued)

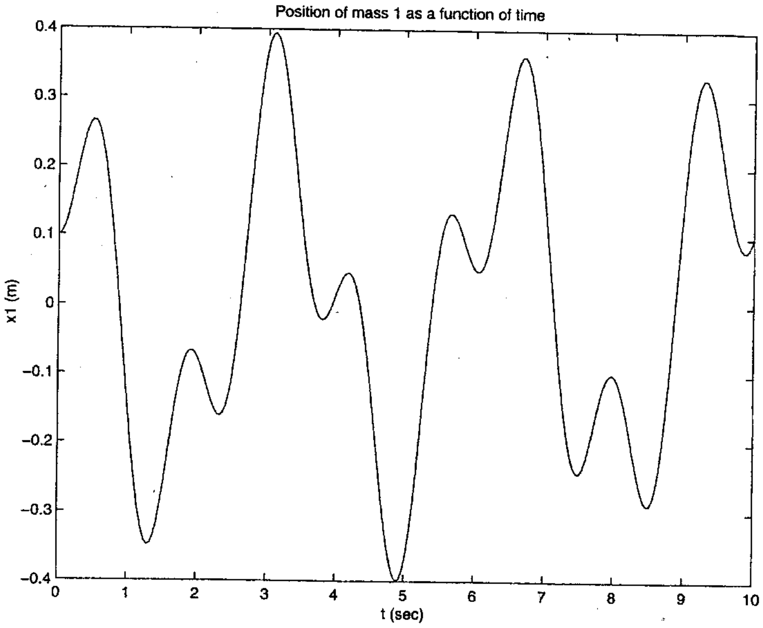
$$(1) \Rightarrow \ddot{x}_1 = -\frac{2k}{m}x_1 + \frac{k}{m}x_2$$

$$(2) \Rightarrow \ddot{x}_2 = \frac{k}{m}x_1 - \frac{k}{m}x_2$$

Let $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$, $z_4 = \dot{x}_2$

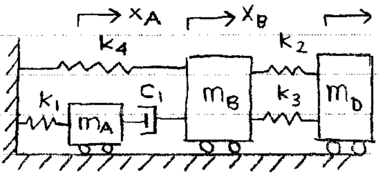
$$\Rightarrow \boxed{\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\frac{2k}{m}z_1 + \frac{k}{m}z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= \frac{k}{m}z_1 - \frac{k}{m}z_3 \end{aligned}} \left.\vphantom{\begin{aligned}a\\a\\a\\a\end{aligned}}\right\} \text{set up for Matlab solution}$$

```
% Matlab solution to problem 5.52:

global m k

m = 1; k = 10;   % arbitrarily chosen parameters
tspan=[0 10];
z0=[0.1;0;0.5;0]; % arbitrary initial conditions
[t,z]=ode45('deriv1',tspan,z0);
x1=z(:,1); v1=z(:,2); x2=z(:,3); v2=z(:,4);

plot(t,x1)        % plot the position of one mass
                  % as a function of time

  function zdot=deriv1(t,z)

  global m k

  zdot=[z(2); -2*k*z(1)/m+k*z(3)/m; z(4);
        k*z(1)/m-k*z(3)/m];
```

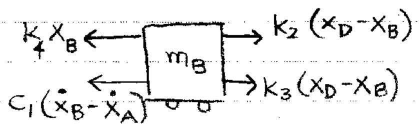


Based on the chosen ICs, the mass should initially move in the $+\hat{i}$-direction and is verified by the above plot. It is seen that the above solution is bounded and not periodic. However, there are "special" ICs which will give rise to periodic solutions (as you will see in Lab #2). One such set is: $x_1(0) = 0.8507\,m$, $x_2(0) = -0.5257\,m$.

5.57



FBD

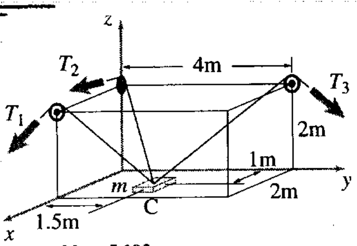


LMB: $\{\sum \underset{\sim}{F} = m_B \underset{\sim}{a}_B\} \cdot \hat{\imath}$

$$\Rightarrow -k_4 x_B - c_1(\dot{x}_B - \dot{x}_A) + (k_2 + k_3)(x_D - x_B) = m_B \ddot{x}_B$$

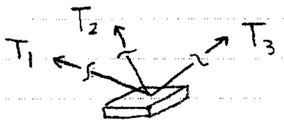
$$\boxed{\begin{aligned} \therefore\ \ddot{x}_B &= \frac{1}{m_B}\left[c_1 \dot{x}_A - c_1 \dot{x}_B - (k_2 + k_3 + k_4) x_B + (k_2 + k_3) x_D\right] \\ \underset{\sim}{a}_B &= \ddot{x}_B\, \hat{\imath} \end{aligned}}$$

5.103



problem 5.103:

FBD



(no gravity)

LMB: $\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$$T_1 \hat{\lambda}_1 + T_2 \hat{\lambda}_2 + T_3 \hat{\lambda}_3 = m\underset{\sim}{a}$$

where
$$\hat{\lambda}_1 = \frac{d\hat{\imath} - \frac{3}{2}d\hat{\jmath} + 2d\hat{k}}{\sqrt{(d)^2 + (-\frac{3}{2}d)^2 + (2d)^2}} = \frac{2}{\sqrt{29}}\left(\hat{\imath} - \tfrac{3}{2}\hat{\jmath} + 2\hat{k}\right)$$

$$\hat{\lambda}_2 = \frac{-d\hat{\imath} - \frac{3}{2}d\hat{\jmath} + 2d\hat{k}}{\sqrt{(-d)^2 + (-\frac{3}{2}d)^2 + (2d)^2}} = \frac{2}{\sqrt{29}}\left(-\hat{\imath} - \tfrac{3}{2}\hat{\jmath} + 2\hat{k}\right)$$

$$\hat{\lambda}_3 = \frac{-d\hat{\imath} + \frac{5}{2}d\hat{\jmath} + 2d\hat{k}}{\sqrt{(-d)^2 + (\frac{5}{2}d)^2 + (2d)^2}} = \frac{2}{3\sqrt{5}}\left(-\hat{\imath} + \tfrac{5}{2}\hat{\jmath} + 2d\hat{k}\right)$$

$$\text{LMB}\cdot\hat{\imath} \Rightarrow T_1\left(\frac{2}{\sqrt{29}}\right) + T_2\left(\frac{-2}{\sqrt{29}}\right) + T_3\left(\frac{-2}{3\sqrt{5}}\right) = ma_x$$

$$\text{LMB}\cdot\hat{\jmath} \Rightarrow T_1\left(\frac{-3}{\sqrt{29}}\right) + T_2\left(\frac{-3}{\sqrt{29}}\right) + T_3\left(\frac{5}{3\sqrt{5}}\right) = ma_y$$

$$\text{LMB}\cdot\hat{k} \Rightarrow T_1\left(\frac{4}{\sqrt{29}}\right) + T_2\left(\frac{4}{\sqrt{29}}\right) + T_3\left(\frac{4}{3\sqrt{5}}\right) = ma_z$$

where $\underset{\sim}{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} = (-.6\hat{\imath} - .2\hat{\jmath} + 2\hat{k})$ m/s²

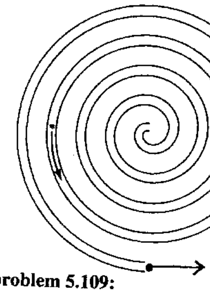
$$\left.\begin{bmatrix} \frac{2}{\sqrt{29}} & \frac{-2}{\sqrt{29}} & \frac{-2}{3\sqrt{5}} \\ -\frac{3}{\sqrt{29}} & -\frac{3}{\sqrt{29}} & \frac{5}{3\sqrt{5}} \\ \frac{4}{\sqrt{29}} & \frac{4}{\sqrt{29}} & \frac{4}{3\sqrt{5}} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} ma_x \\ ma_y \\ ma_z \end{bmatrix}\right\} \text{"}Ax = b\text{"}$$

(continued)

```
% Matlab solution to problem 5.103:

m = 2;                          % units of kg
acc = [-0.6 ; -0.2 ; 2];        % units of m/s^2

% Solve Ax=b where A is the coefficient matrix,
% x=[T1,T2,T3] is the vector of tensions, and
% b is the mass times acceleration.

b = m * acc;
A = [2/sqrt(29) -2/sqrt(29) -2/(3*sqrt(5));
    -3/sqrt(29) -3/sqrt(29)  5/(3*sqrt(5));
     4/sqrt(29)  4/sqrt(29)  4/(3*sqrt(5))];

tension=A\b

% Matlab output:
```

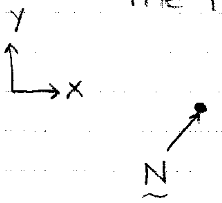
```
tension =

T1= 1.0770          % in units of Newtons
T2= 2.5580
T3= 2.1802
```

5.109



problem 5.109:

FBD (while in the tube)



(no gravity)

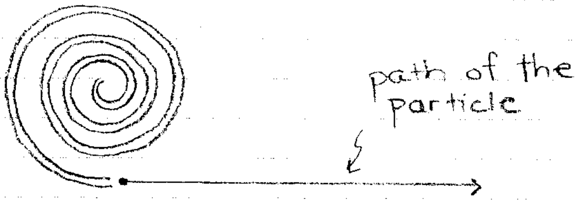
The normal force of the tube on the particle keeps the particle moving in a spiral path.

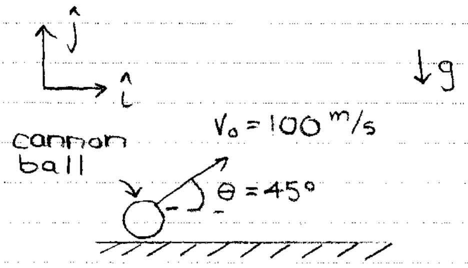
LMB: $\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = \underset{\sim}{0}$

FBD (when expelled)

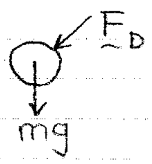
(frictionless, horizontal table)

Since no forces act on the particle, it will move in a straight line (Newton's 1^{ST} law).





FBD



$\underset{\sim}{F}_D = -C|\underset{\sim}{v}|\cdot\frac{\underset{\sim}{v}}{|\underset{\sim}{v}|}$ ← drag force is proportional to speed (direction $-\underset{\sim}{v}/|\underset{\sim}{v}|$ opposes the motion)

LMB: $\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$-mg\hat{j} - C\underset{\sim}{v} = m\underset{\sim}{a}$

where $\underset{\sim}{v} = \dot{x}\hat{\imath} + \dot{y}\hat{j}$, $\underset{\sim}{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{j}$

LMB $\cdot \hat{\imath} \Rightarrow -C\dot{x} = m\ddot{x}$

LMB $\cdot \hat{j} \Rightarrow -mg - C\dot{y} = m\ddot{y}$

Let $z_1 = x$, $z_2 = \dot{x}$, $z_3 = y$, $z_4 = \dot{y}$

$$\Rightarrow \left.\boxed{\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\frac{c}{m} z_2 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= -\frac{c}{m} z_4 - g \end{aligned}}\right\} \text{set up for Matlab solution}$$

```
% Matlab solution to problem 5.113:

global c m g

c = 5; g = 10; m = 10; theta = pi/4; speed = 100;
x0 = 0; y0 = 0; v0=speed*[cos(theta); sin(theta)];
tspan=[0 9];
z0=[x0; v0(1); y0; v0(2)];
[t,z]=ode45('deriv2',tspan,z0);
x=z(:,1); y=z(:,3);

% Plot the numerical solution.
plot(x,y)
axis('equal')
xlabel('x (m)'); ylabel('y (m)')
title('Trajectory of the cannonball')
hold on

% Plot the hand solution.
k1=m*z0(2)/c; k2=(m/c)*(z0(4)+m*g/c);
xh=-m*z0(2)*exp(-c*t/m)/c+k1;
yh=-(m/c)*(z0(4)+m*g/c)*exp(-c*t/m)-m*g*t/c+k2;
plot(xh,yh,':')
legend('numerical','hand')
```

(continued) →

```
function zdot=deriv2(t,z)

global c m g

zdot=[z(2); -c * z(2)/m; z(4); -c * z(4)/m - g];
```

• Solution by hand:

(1) $\Rightarrow \quad m\frac{du}{dt} = -Cu$ where $u = \dot{x}$

$\therefore \quad u = u_0 e^{-\frac{c}{m}t}$ OR $\frac{dx}{dt} = \dot{x}_0 e^{-\frac{c}{m}t}$

$$\Rightarrow \boxed{x(t) = \frac{-m\dot{x}_0}{c} e^{-\frac{c}{m}t} + K_1}$$

(2) $\Rightarrow \quad m\frac{dv}{dt} = -Cv - mg$ where $v = \dot{y}$

$$\therefore \int_{v_0}^{v} \frac{dv'}{v' + \frac{mg}{c}} = \int_{t_0}^{t} -\frac{c}{m}\,dt'$$

$$\ln\left(v' + \frac{mg}{c}\right)\Big|_{v_0}^{v} = -\frac{c}{m}t \qquad (t_0 = 0)$$

$$\Rightarrow \ln\left(\frac{v + mg/c}{v_0 + mg/c}\right) = -\frac{c}{m}t$$

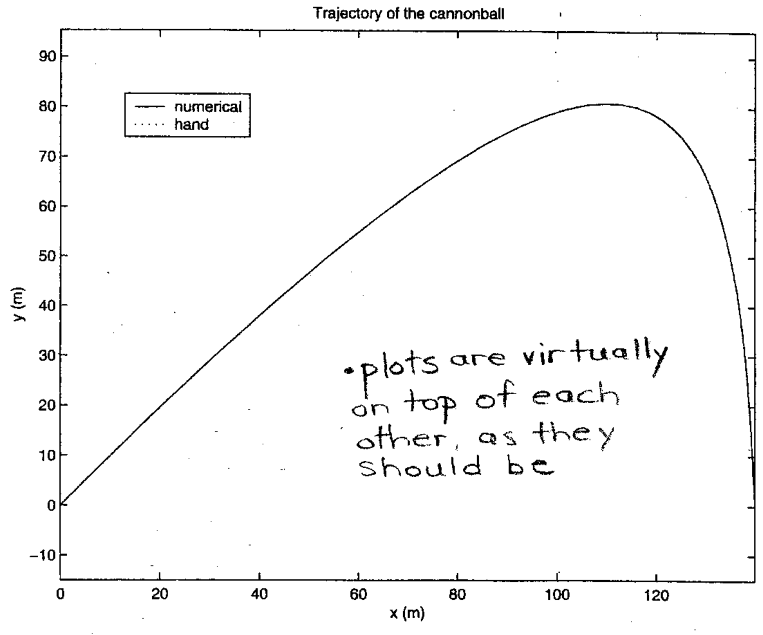
$$\therefore \quad v = \frac{dy}{dt} = (v_0 + mg/c)e^{-\frac{c}{m}t} - \frac{mg}{c}$$

$$\Rightarrow \boxed{y(t) = -\frac{m}{c}\left(\dot{y}_0 + \frac{mg}{c}\right)e^{-\frac{c}{m}t} - \frac{mg}{c}t + K_2}$$

Let the initial position be given by $\underset{\sim}{r}_0 = x_0\hat{i} + y_0\hat{j} = \underset{\sim}{0}$ with initial velocity:

$$\underset{\sim}{v}_0 = \dot{x}_0\hat{i} + \dot{y}_0\hat{j} = (100\,\text{m/s})(\cos 45°\hat{i} + \sin 45°\hat{j})$$

With $x_0 = 0$, $y_0 = 0$ the constants K_1, K_2 are: $K_1 = \frac{m\dot{x}_0}{c}$, $K_2 = \frac{m}{c}\left(\dot{y}_0 + \frac{mg}{c}\right)$



• plots are virtually on top of each other, as they should be