T&AM 203 - DYNAMICS

HW #2 - DUE 2/8/00

JOE BURNS

Motion of ball with air drag

① $g \downarrow$ $bv^2 \uparrow$, m, $mg \downarrow$, $y \downarrow$

ⓐ $\dot{\underline{L}} = \Sigma \underline{F}$

$$m\ddot{y} = mg - b\dot{y}^2$$

$$\boxed{\ddot{y} = g - \frac{b}{m}\dot{y}^2} \quad (1a)$$

$\dot{y} = v$ or

$$\boxed{\dot{v} = g - \frac{b}{m}v^2} \quad (1b)$$

ⓑ Constant speed motion occurs when

$$\dot{v} = 0 \Rightarrow g = \frac{b}{m}v^2_{Terminal}$$

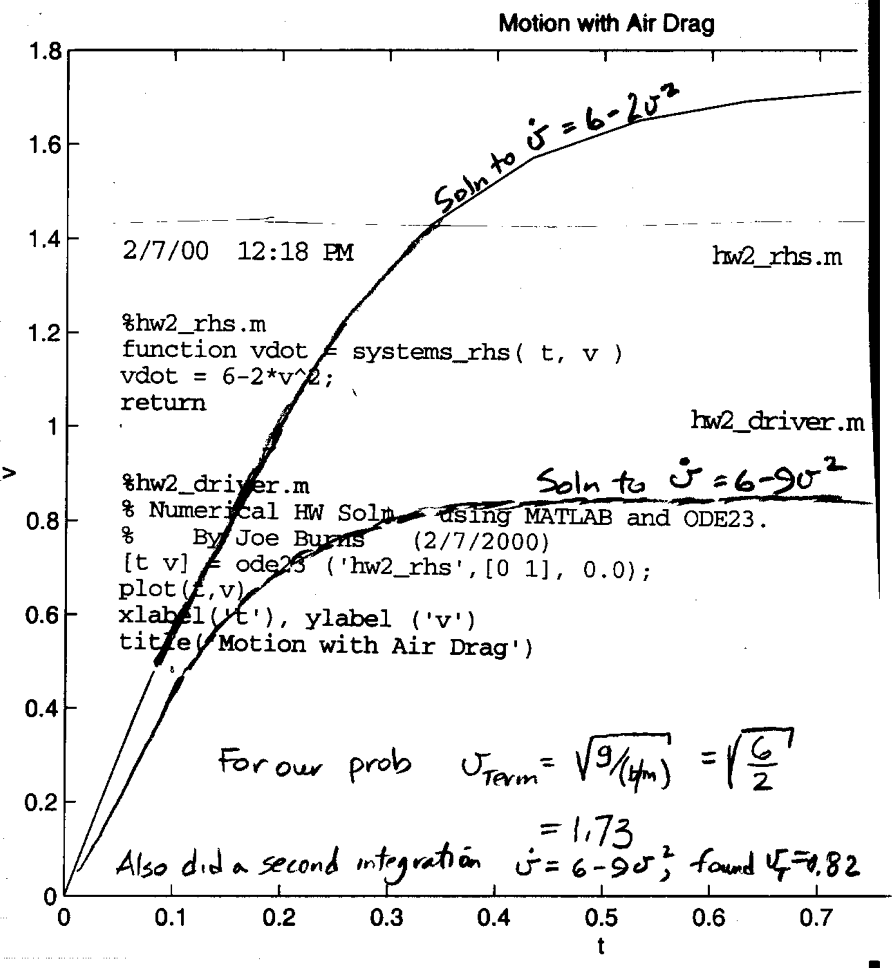
$$\boxed{v_{Term} = \sqrt{\frac{mg}{b}}}$$

Soln to ODE

$$\frac{dv}{g - b/_m v^2} = dt \Rightarrow t = \frac{m}{b}\int_0^v \frac{dv}{\frac{mg}{b} - v^2}$$

$$\frac{\sqrt{\frac{mg}{b}} + v}{\sqrt{\frac{mg}{b}} - v} = e^{2\sqrt{\frac{gb}{m}}t}$$

ⓒ We're only interested in speed as function of time, so we solve (1b) numerically. It is a first-order ODE with const coef. Non-homogeneous (due to g) and non-linear (due to v^2)



2/7/00 12:18 PM hw2_rhs.m

```
%hw2_rhs.m
function vdot = systems_rhs( t, v )
vdot = 6-2*v^2;
return
```

hw2_driver.m

```
%hw2_driver.m
% Numerical HW Soln - using MATLAB and ODE23.
%     By Joe Burns    (2/7/2000)
[t v] = ode23 ('hw2_rhs',[0 1], 0.0);
plot(t,v)
xlabel('t'), ylabel ('v')
title('Motion with Air Drag')
```

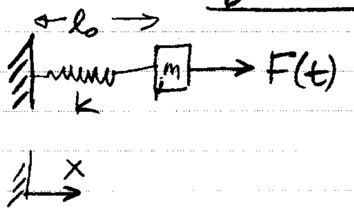
For our prob $v_{Term} = \sqrt{g/(b/m)} = \sqrt{\frac{6}{2}}$

$= 1.73$

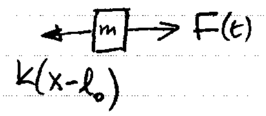
Also did a second integration $\dot{v} = 6 - 9v^2$; found $v_T = 0.82$

Eqns of motion for mass-spring systems

a.)



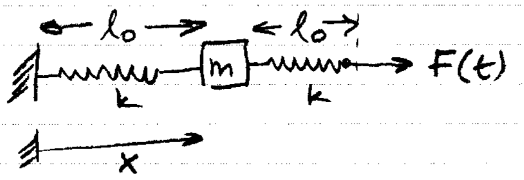
$\hat{\imath}\cdot\left(\Sigma \underline{F} = \dot{\underline{L}}\right)$



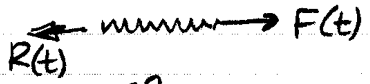
$F(t) - k(x-\ell_o) = m\ddot{x}$

$$\boxed{\ddot{x} + \frac{k}{m}x = \frac{F + k\ell_o}{m}}$$

b.)



Look first at FBD of RH spring



$\dot{L}_{spring} = \frac{d}{dt}\cancel{m}\overset{0}{v} = \Sigma \underline{F}$

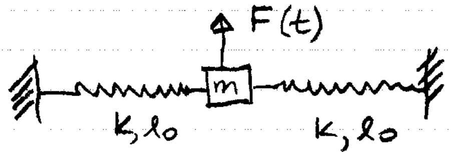
$\therefore -R(t) + F(t) = 0$

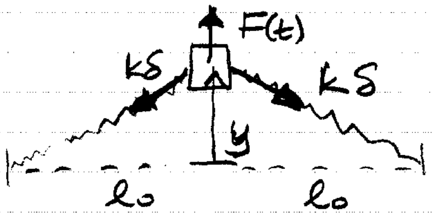
$R(t) = F(t)$

So the inclusion of RH spring is inconsequential. This problem is just a).

$$\boxed{\ddot{x} + \frac{k}{m}x = \frac{F + k\ell_o}{m}}$$

©





when mass is displaced by y

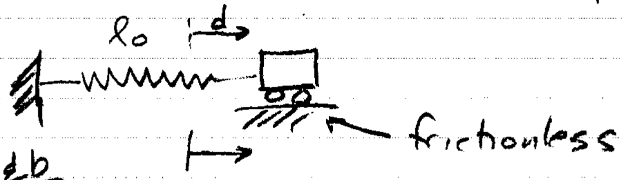
$$\delta = \underbrace{(\ell_o^2 + y^2)^{1/2}}_{\text{length of stretched spring}} - \underbrace{\ell_o}_{\text{unstretched}} = \ell_o\left(1 + \frac{y^2}{\ell_o^2}\right)^{1/2} - \ell_o$$

$$\delta = \ell_o\left(1 + \frac{1}{2}\frac{y^2}{\ell_o^2} + \cdots\right) - \ell_o = \frac{1}{2}\frac{y^2}{\ell_o}$$

$$\left(\dot{\underline{L}} = \Sigma \underline{F}\right)\cdot\hat{\jmath} = \quad m\ddot{y} = -2k\left(\frac{1}{2}\frac{y^2}{\ell_o}\right)\frac{y}{(\ell_o^2+y^2)^{1/2}} \approx -\frac{ky^3}{\ell_o^2}$$

$$\boxed{\ddot{y} = -\frac{k}{m}\frac{y^3}{\ell_o}}$$

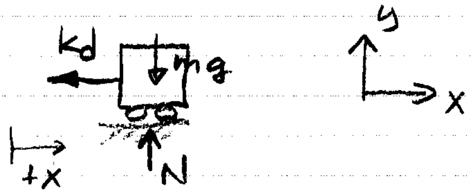
Mass released from rest with spring



a & b

Measure positions from unstretched

FBD



$$\hat{\imath}\cdot\left(\Sigma \underline{F} = \dot{\underline{L}}\right)$$

$$-kx = m\ddot{x} \Rightarrow \boxed{\ddot{x} + \frac{k}{m}x = 0} \quad *$$

acceleration just after release is $\boxed{-\frac{kd}{m}}$

ⓒ Solution to $*$ is

$$x = A\cos\sqrt{\tfrac{k}{m}}\,t + B\sin\sqrt{\tfrac{k}{m}}\,t$$

Apply initial conditions

$$x(0) = d \;,\; \dot{x}(0) = 0$$

$$x(0) = d = A(1) + B(0) = A$$

$$\dot{x}(0) = 0 = -A\sqrt{\tfrac{k}{m}}\,(0) + B\sqrt{\tfrac{k}{m}}\,(1)$$

$$\therefore\; A = d \;,\; B = 0$$

$$\boxed{x(t) = d\cos\sqrt{\tfrac{k}{m}}\,t}$$

ⓓ Speed @ $x = 0$

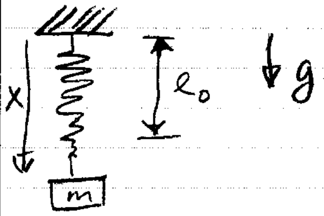
$$\dot{x}(t) = -d\sqrt{\tfrac{k}{m}}\,\sin\sqrt{k/m}\,t$$

What t is $x = 0$? $\quad x(t) = 0 \Rightarrow \cos\sqrt{\tfrac{k}{m}}\,t = 0$

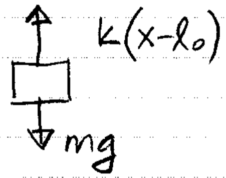
At these times $\sin\sqrt{\tfrac{k}{m}}\,t = \pm 1$

$$\therefore \boxed{|\dot{x}(t)| = d\sqrt{k/m}}$$

Mass hangs from spring



(a) FBD



(b) $\left(\Sigma \underline{F} = \dot{\underline{L}}\right)\cdot\hat{\imath}$

$$\boxed{mg - k(x-\ell_o) = m\ddot{x}}$$

(c) $$\boxed{\ddot{x} + \frac{k}{m}x = \frac{k}{m}\ell_o + g} \quad *$$

(d) Try $x(t) = X = \ell_o + mg/k$

$$\ddot{X} = 0$$

$$\therefore \ddot{X} + \frac{k}{m}X = 0 + \frac{k}{m}\left(\ell_o + \frac{mg}{k}\right) = \frac{k\ell_o}{m} + g,$$

which is RHS of $*$

$\therefore$ X satisfies $*$

(e) This solution says that the mass is stationary; it hangs $\ell_o + mg/k$ below ////.

ℓ_o: unstretched length; mg/k: stretch under weight mg

(f) Use $x = \hat{x} + \left(\ell_o + \frac{mg}{k}\right)$ in $*$

$$\ddot{\hat{x}} + \frac{k}{m}\left(\hat{x} + \ell_o + \frac{mg}{k}\right) = \frac{k\ell_o}{m} + g$$

$$\ddot{\hat{x}} + \frac{k}{m}\hat{x} + \cancel{\frac{k\ell_o}{m}} + \cancel{g} = \cancel{\frac{k\ell_o}{m}} + \cancel{g}$$

$$\boxed{\ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0}$$

Harmonic Oscillator about stretched equilibrium

We solve the (f) eqn. rather than (c) because it's homogeneous

(g) Released from rest at $x = D$ or from

$$\hat{x} = D - \left(\ell_o + \frac{mg}{k}\right)$$

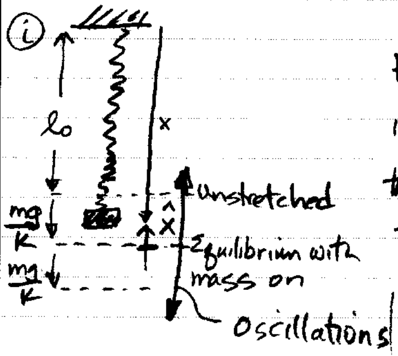
Soln is $$\boxed{\hat{x} = \left(D - \ell_o + \frac{mg}{k}\right)\cos\sqrt{\frac{k}{m}}\,t}$$

by analogy to part c of prob 3

(Continued)

(h) The period comes from $\sqrt{\frac{k}{m}}\,T = 2\pi \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

(i)



For $D > l_o + 2mg/k$, the initial stretch in spring is more than twice that due to the weight. Thus oscillations will carry weight back beyond unstretched length. Spring will then buckle and equation of motion will no longer be valid.

PROB. 5 (CONT'D)

Time of flight $= t_f = \frac{2v_o}{g}$

$= \frac{2\sqrt{2gh}}{g} = \sqrt{\frac{8h}{g}} = \sqrt{\frac{8(5)\text{ ft.}}{32.2\text{ ft/sec}^2}} = 1.11\text{ sec}$

This comes from either i.) solving * or ii.) Saying t_f is twice the time for gravity to slow v_o to zero

Time on trampoline

To get the time in contact w/ trampoline, we find elapsed time from first contact until max extension, where $\dot{y} = 0$, and double it. For this, we need a gen'l solution

v_o --- level

y

$\dot{y} = 0$

$$y = \frac{mg}{k} + A\cos\omega t + B\sin\omega t \quad ⊗ \qquad \omega = \sqrt{k/m}$$

I.C. $y = 0,\ \dot{y} = v_o$ @ $t = 0$

$y(0) = 0 = \frac{mg}{k} + A + 0 \Rightarrow A = -mg/k$

$\dot{y}(0) = v_o = B\omega \Rightarrow B = v_o/\omega$

} Plugging into ⊗

Thus $\boxed{y = \frac{mg}{k}(1 - \cos\omega t) + v_o/\omega\, \sin\omega t}$

This assumes the trampoline is massless & has no inertia so it is horizontal except when person is in contact.

$\dot{y} = 0$ at bottom of motion. Differentiate y(t) in □

$$\dot{y} = \frac{g}{\omega^2}(\omega \sin\omega t + v_o \cos\omega t)$$

$$\dot{y}(T_{tramp}) = 0 = \frac{g}{\omega}\sin\omega T_{tramp} + v_o\cos\omega T_{tramp}$$

$$\therefore \tan\omega T_{Tramp} = -\frac{\omega v_o}{g}$$

To get down: or $T_{tramp} = \sqrt{\frac{m}{k}}\tan^{-1}\left(\frac{-v_o\sqrt{k/m}}{g}\right)$

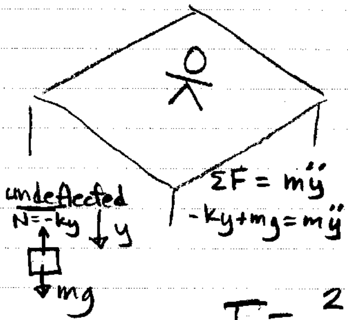
So round-trip

$$2T_T = 2\sqrt{\frac{m}{k}}\tan^{-1}\sqrt{\frac{2gh\,k}{g^2 m}}$$

$$= 2\left(\frac{150\text{ lbm}}{200\,\frac{\text{lbf}}{\text{ft}}\cdot 32.2\frac{\text{ft}}{\text{sec}^2}}\right)^{1/2}\tan^{-1}\left(-\left[\frac{2(5\text{ft})\,200\,\frac{\text{lbf}}{\text{ft}}}{150\text{ lbm}\cdot g}\right]^{1/2}\right) = 0.30(\pi - 1.50)\text{sec} = 0.49\text{ sec}$$

Total elapsed time $= t_f + 2T_t = 1.11\text{ sec} + 0.49\text{ sec} = 1.60\text{ sec}$

Girl on Trampoline



$\Sigma F = m\ddot{y}$

$-ky + mg = m\ddot{y}$

ⓐ As long as feet don't leave the trampoline, then this is just a vibrating mass-spring $\ddot{y} + \frac{k}{m} y = g$

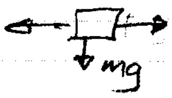
$$T_{vibe} = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{150 \text{ lbm}}{200 \text{ lbf/ft} \cdot 32.2 \text{ ft/sec}^2}}$$

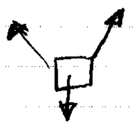
$$\boxed{T_{vibe} = 0.96 \text{ sec}}$$

ⓑ

Equilibrium

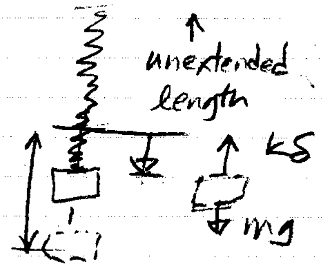


Top of flight (trampoline is level)



Max Extension

In terms of spring model, these are



$$\delta = \frac{mg}{k}$$

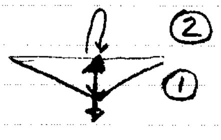
$$\delta = \frac{150 \text{ lbm} \cdot 32.2 \text{ ft/sec}^2}{200 \cdot 32 \frac{\text{lbm}}{\text{ft}}\frac{\text{ft}}{\text{sec}^2}} = \frac{3}{4} \text{ FT.}$$

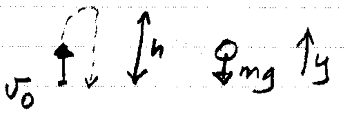
$1 \text{ lbf} = 1 \text{ lbm} \cdot g = 32.2 \text{ lbm (ft/s}^2)$

ⓒ The trampolinist's motion ~~has two~~ parts:

① Up & down motion on trampoline; this takes $< T_{vibe}$ because a complete cycle is not completed. See ←.

② The trajectory through the air





$\cancel{m}\ddot{y} = -\cancel{m}g$

Integrating twice $y = \cancel{y_0}^{\,0} + v_0 t - \frac{1}{2}gt^2$

$* \boxed{y = (v_0 - \frac{1}{2}gt)t}$

Returns at $v_0 - \frac{1}{2}gt = 0 \Rightarrow t = \frac{2v_0}{g}$

By $v^2 = v_0^2 - 2gy \Rightarrow$ At top of path $\boxed{0 = v_0^2 - 2gh}$

$v_0 = \sqrt{2gh}$

← CONTINUED