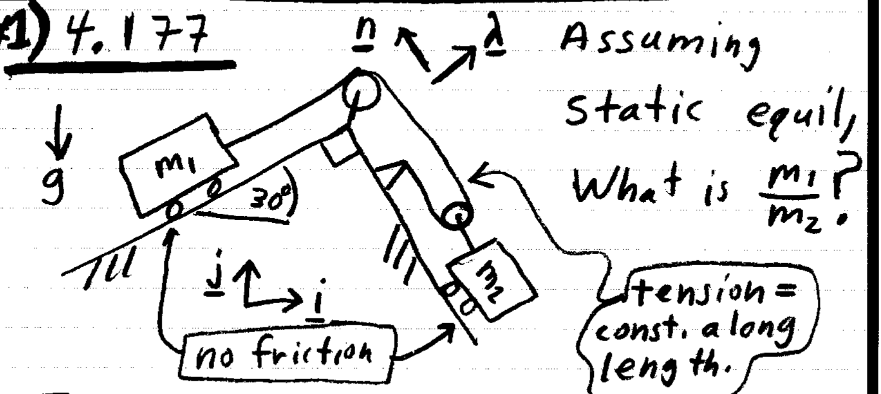
TAM 203, HW 1

due Feb 1, 2000

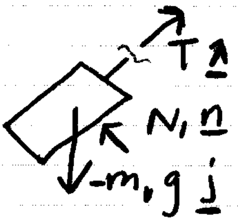
"Solutions" - A. Ruina

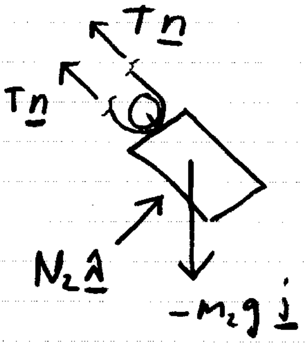
#1) 4.177



Assuming static equil, What is $\frac{m_1}{m_2}$?

FBDs





$$\Sigma \underline{F} = \underline{0}$$

$$\Rightarrow \{-m_1 g \underline{j} + N_1 \underline{n} + T\underline{\lambda} = \underline{0}\}$$

$\{\ \}\cdot\underline{\lambda} \Rightarrow$

$$-m_1 g \overbrace{\underline{j}\cdot\underline{\lambda}}^{1/2} + N_1 \overbrace{\underline{n}\cdot\underline{\lambda}}^{0} + T \overbrace{\underline{\lambda}\cdot\underline{\lambda}}^{1} = \overbrace{\underline{0}\cdot\underline{\lambda}}^{0}$$

$$\Rightarrow \tfrac{1}{2} m_1 g = T \qquad (1)$$

$$\Sigma \underline{F} = \underline{0}$$

$$\{-m_2 g \underline{j} + N_2 \underline{\lambda} + 2T\underline{n} = \underline{0}\}$$

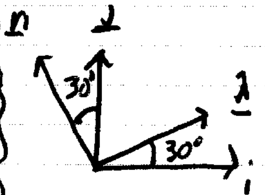
$\{\ \}\cdot\underline{n} \Rightarrow$

$$-m_2 g \underbrace{\underline{j}\cdot\underline{n}}_{\sqrt{3}/2} + N_2 \overbrace{\underline{\lambda}\cdot\underline{n}}^{0} + 2T \overbrace{\underline{n}\cdot\underline{n}}^{1} = \underline{0}\cdot\underline{n}$$

$$\Rightarrow \frac{\sqrt{3}}{2} m_2 g = 2T \quad (2)$$

$$(1),(2) \Rightarrow \frac{m_1}{m_2} = \frac{2T/g}{4T/\sqrt{3}g} \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{\sqrt{3}}{2}}$$

Geometry $\underline{i}, \underline{j}, \underline{\lambda}, \underline{n}$ are unit vectors

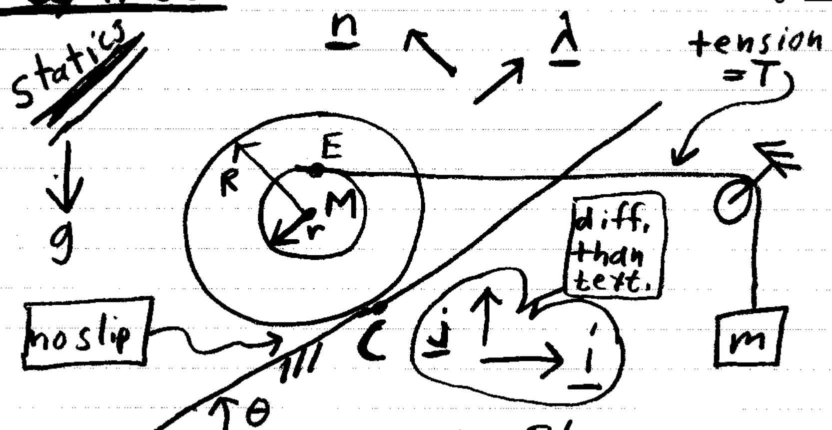


$$\underline{i}\cdot\underline{i} = \underline{j}\cdot\underline{j} = \underline{\lambda}\cdot\underline{\lambda} = \underline{n}\cdot\underline{n} = 1$$

$$\underline{i}\cdot\underline{j} = \underline{\lambda}\cdot\underline{n} = 0$$

$$\underline{j}\cdot\underline{\lambda} = \sin 30^\circ = 1/2$$

$$\underline{j}\cdot\underline{n} = \cos 30^\circ = \sqrt{3}/2$$



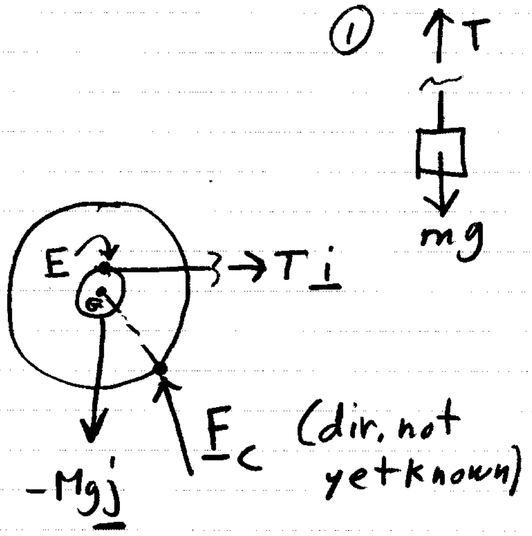
$r = R/2$

Find m/M, T, reaction at C etc.

FBDs

(1)

(2)



$\underline{r}_{CG} = R\,\underline{n}$

$\underline{r}_{CE} = R\underline{n} + r\underline{j}$

FOR FBD (1): $\{\sum \underline{F} = \underline{0}\} \cdot \underline{j} \Rightarrow \boxed{T = mg}$

FOR FBD (2):

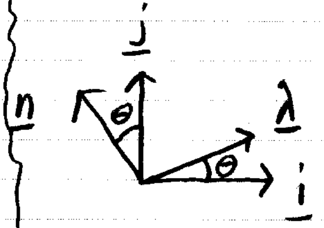
$$\sum \underline{M}_{/C} = \underline{0}$$

$$\Rightarrow \underline{r}_{CG} \times (-Mg\underline{j}) + \underline{r}_{CE} \times (T\underline{i}) = \underline{0}$$

$$(R\underline{n}) \times (-Mg\,\underline{j}) + (R\underline{n} + r\underline{j}) \times (T\underline{i}) = \underline{0}$$
(1)

cont'd →

Geometry



$\underline{\lambda} = \cos\theta\, \underline{i} + \sin\theta\, \underline{j}$

$\underline{n} = -\sin\theta\, \underline{i} + \cos\theta\, \underline{j}$

$\underline{n} \times \underline{j} = -\sin\theta\, \underline{k}$

$\underline{i} \times \underline{j} = \underline{\lambda} \times \underline{n} = \underline{k}$ $\qquad \underline{n} \times \underline{i} = -\cos\theta\, \underline{k}$

etc.

$$(1) \Rightarrow \left\{ -RMg(-\sin\theta\, \underline{k}) + RT(-\cos\theta\, \underline{k}) + rT(-\underline{k}) = \underline{0} \right\}$$

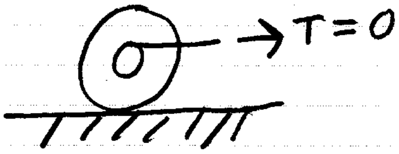
$$\{\ \}\cdot \underline{k} \Rightarrow RMg\sin\theta - RT\cos\theta - rT = 0$$

$$\frac{r}{R} = \frac{1}{2},\ T = mg \Rightarrow M\cancel{g}\sin\theta - m\cancel{g}\left(\cos\theta + \frac{1}{2}\right) = 0$$

$$\Rightarrow \boxed{\frac{m}{M} = \frac{\sin\theta}{\cos\theta + 1/2}} \quad (a)$$

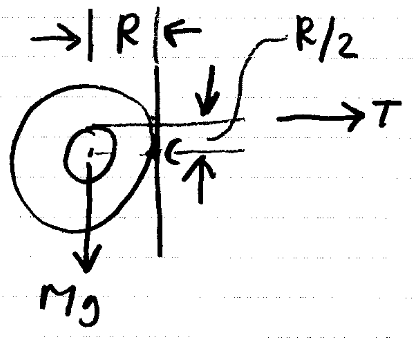
Checks:

$\theta = 0$:



formula gives 0 as it should

$\theta = \pi/2$:



tension has half the lever arm about pt. c $\Rightarrow$ $T = 2Mg \Rightarrow m = 2M$.

Formula above gives $\frac{m}{M} = 2$ as it should.

(cont'd)

b) tension?

$$\boxed{T = mg = \frac{2Mg\sin\theta}{2\cos\theta + 1}} \quad (b)$$

(Note: no R in answer above.)

c) Reaction at C $\underline{F}_c = ?$

Go back to FBD ①

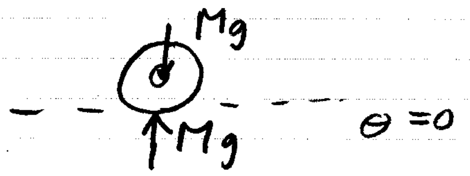
$$\Sigma \underline{F}_i = \underline{0}$$

$$\Rightarrow -Mg\underline{j} + \underbrace{T}_{mg}\underline{i} + \underline{F}_c = \underline{0}$$

$$\Rightarrow \boxed{\underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}\underline{i} + \underline{j}\right]} \quad (c)$$

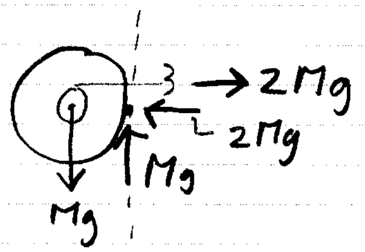
Checks

$\theta = 0$



$\underline{F}_c = Mg\underline{j}$ (as per formula)

$\theta = \pi/2$

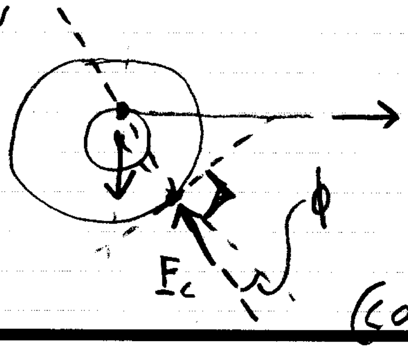


formula gives $\underline{F}_c = Mg(-2\underline{i} + \underline{j})$

as per FBD above

d) Use 3-force body logic

All 3 forces must have lines of action which intersect at E.



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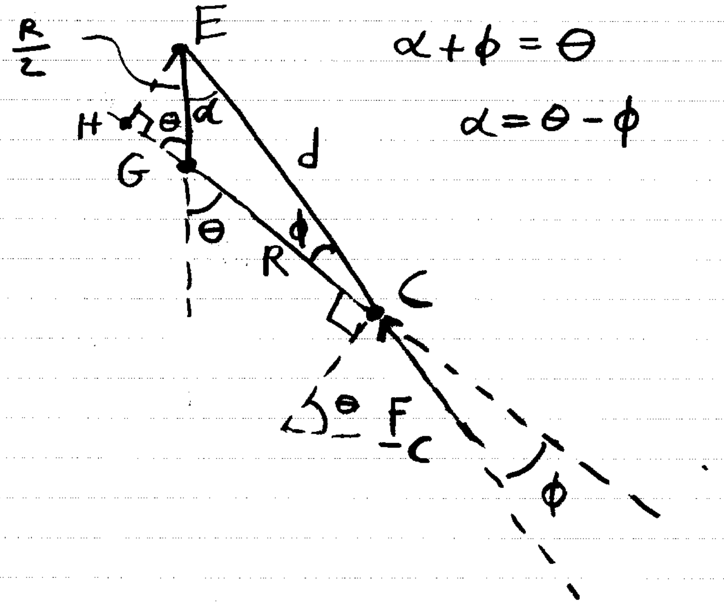


$\alpha + \phi = \theta$

$\alpha = \theta - \phi$

Look at triangle CEH:

$$\tan\phi = \frac{\frac{R}{2}\sin\theta}{R + \frac{R}{2}\cos\theta} = \frac{\sin\theta}{2+\cos\theta} \quad (*)$$

Now look at the soln. to part (c)

since $\underline{i} = \cos\theta\,\underline{\lambda} - \sin\theta\,\underline{n}$

and $\underline{j} = \sin\theta\,\underline{\lambda} + \cos\theta\,\underline{n}$

$$\Rightarrow \underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}\left(\cos\theta\,\underline{\lambda} - \sin\theta\,\underline{n}\right) + \left(\sin\theta\,\underline{\lambda} + \cos\theta\,\underline{n}\right)\right]$$

$$= Mg\left[\frac{-2\sin\theta\cos\theta + (2\cos\theta+1)\sin\theta}{2\cos+1}\underline{\lambda} + \frac{+2\sin^2\theta + \cos\theta(2\cos\theta+1)}{2\cos\theta+1}\underline{n}\right]$$

$$= Mg\left[\frac{\sin\theta}{2\cos\theta+1}\underline{\lambda} + \frac{2+\cos\theta}{2\cos\theta+1}\underline{n}\right]$$

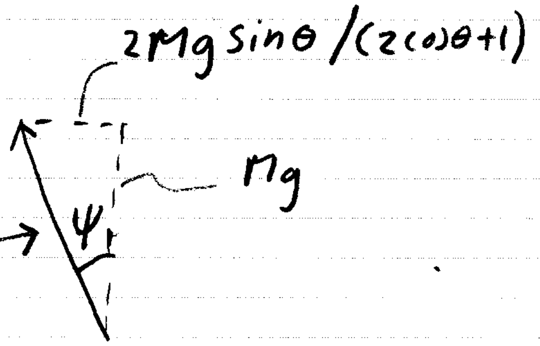
$$\Rightarrow \tan\phi = \frac{\left(\frac{\sin\theta}{2\cos\theta+1}\right)}{\left(\frac{2+\cos\theta}{2\cos\theta+1}\right)} = \frac{\sin\theta}{2+\cos\theta} \quad (**)$$

which checks w/ $*$ above

(cont'd)

e) recall from (c)

$$\underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}\,\underline{i} + \underline{j}\right]$$

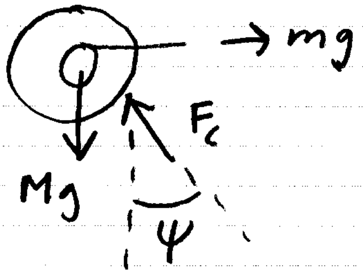


$$\Rightarrow \tan\psi = \frac{2\sin\theta}{2\cos\theta+1}$$

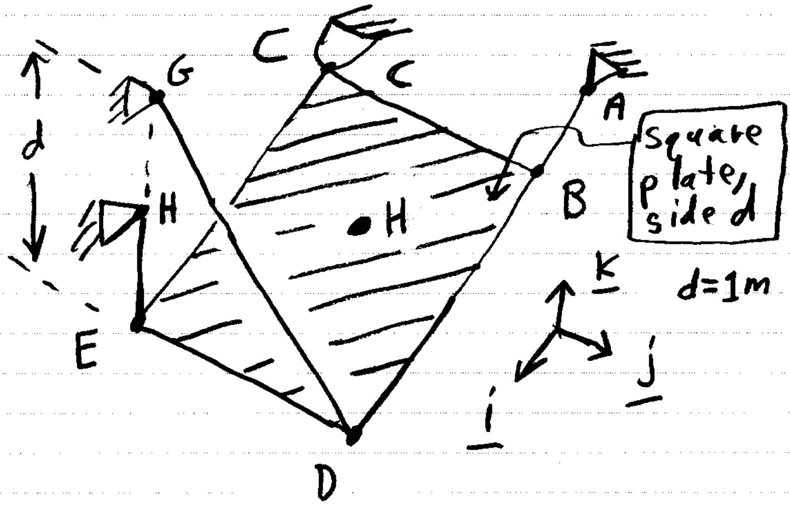
from part b: $\frac{m}{M} = \frac{2\sin\theta}{2\cos\theta+1}$

$$\Rightarrow \boxed{\tan\psi = \frac{m}{M}} \quad (e)$$

Note: despite all the algebra, this answer could be found directly from this FBD:

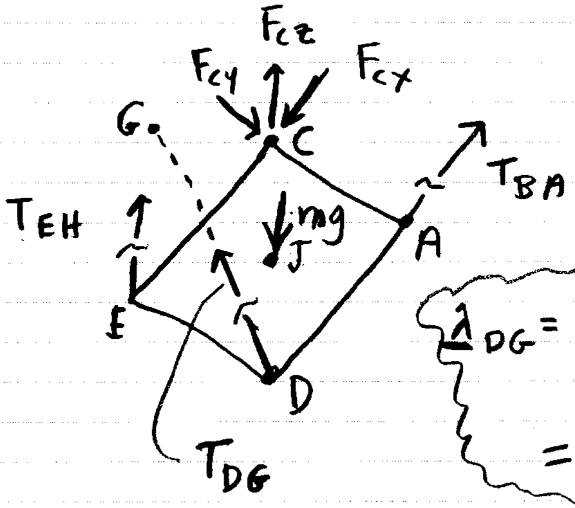


3) 4.188



Find the various tensions & reactions.

a) FBD



$$\underline{\lambda}_{DG} = \frac{\underline{r}_{DG}}{|\underline{r}_{DG}|} = \frac{-\underline{j}+\underline{k}}{\sqrt{2}}$$

b) Consider the axis DE. The only forces w/ nonzero moment are mg at H and F_{cz} at C. So

$$\boxed{F_{cz} = mg/2}$$

c) $\Sigma \underline{F} = \underline{0}$

$$\Rightarrow F_{cx}\underline{i} + F_{cy}\underline{j} + F_{cz}\underline{k}$$
$$+ T_{EH}\underline{k}$$
$$+ T_{BA}(-\underline{i})$$
$$+ T_{DG}\underline{\lambda}_{DG} \quad - mg\underline{k} = \underline{0}$$

$$\underline{\lambda}_{DG} = \frac{-\underline{j}+\underline{k}}{\sqrt{2}}$$

(cont'd)

d) $\sum \underline{M}_{/J} = \underline{0}$

$$\underline{r}_{C/J} \times \underline{F}_C + \underline{r}_{D/J} \times (T_{DG} \underline{\lambda}_{DG})$$
$$+ \underline{r}_{E/J} \times (T_{EH} \underline{k}) + \underline{r}_{A/J} \times (T_{BA}(-\underline{i}))$$
$$= \underline{0}$$

Factoring out a common factor of $d/2$ (.5m)

$$\Rightarrow (-\underline{i} - \underline{j}) \times (F_{Cx}\underline{i} + F_{Cy}\underline{j} + F_{Cz}\underline{k})$$
$$+ (\underline{i} - \underline{j}) \times (T_{EH}\underline{k}) + (\underline{i} + \underline{j}) \times T_{DG}\frac{(-\underline{j}+\underline{k})}{\sqrt{2}}$$
$$+ (-\underline{i} + \underline{j}) \times (T_{BA}(-\underline{i}))$$
$$= \underline{0}$$

$$\Rightarrow \left[(\underline{k})F_{Cx} + (-\underline{k})F_{Cy} + (\underline{j} - \underline{i})F_{Cz}\right]$$
$$+ \left[(-\underline{j} - \underline{i})T_{EH}\right] + \left[(\underline{i} - \underline{j} - \underline{k})\frac{T_{DG}}{\sqrt{2}}\right]$$
$$+ \underline{k}\, T_{BA} = \underline{0}$$

e) Write eqs. in comp. form.

$\sum \underline{F} = \underline{0} \Rightarrow$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}F_{Cx} + \begin{bmatrix}0\\1\\0\end{bmatrix}F_{Cy} + \begin{bmatrix}0\\0\\1\end{bmatrix}F_{Cz}$$
$$+ \begin{bmatrix}0\\0\\1\end{bmatrix}T_{EH} + \begin{bmatrix}-1\\0\\0\end{bmatrix}T_{BA} + \begin{bmatrix}0\\-\sqrt{2}/2\\\sqrt{2}/2\end{bmatrix}T_{DG}$$
$$= mg\begin{bmatrix}0\\0\\1\end{bmatrix}$$

1st comp, 2nd comp, 3rd comp make up 3 eqs in 6 unknowns.

(cont'd)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\sqrt{2}/2 \\ 0 & 0 & 1 & 1 & 0 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} F_{cx} \\ F_{cy} \\ F_{cz} \\ T_{EH} \\ T_{BA} \\ T_{DG} \end{bmatrix} = mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (*)$$

Likewise for moments ...

$$\sum \underline{M}_{/J} = \underline{0}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} F_{cx} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} F_{cy} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} F_{cz}$$

$$+ \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} T_{EH} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T_{BA} + \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix} T_{DG}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & -1 & -1 & 0 & \sqrt{2}/2 \\ 0 & 0 & 1 & -1 & 0 & -\sqrt{2}/2 \\ 1 & -1 & 0 & 0 & 1 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} F_{cx} \\ F_{cy} \\ F_{cz} \\ T_{EH} \\ T_{BA} \\ T_{DG} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (**)$$

Putting $(*)$ & $(**)$ together and defining $\hat{\ }$ variables as $\hat{(\ )} = \frac{(\ )}{mg}$ we get

(cont'd)

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\sqrt{2}/2 \\ 0 & 0 & 1 & 1 & 0 & \sqrt{2}/2 \\ 0 & 0 & -1 & -1 & 0 & \sqrt{2}/2 \\ 0 & 0 & 1 & -1 & 0 & -\sqrt{2}/2 \\ 1 & -1 & 0 & 0 & 1 & -\sqrt{2}/2 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \hat{F}_{cx} \\ \hat{F}_{cy} \\ \hat{F}_{cz} \\ \hat{T}_{EH} \\ \hat{T}_{BA} \\ \hat{T}_{DG} \end{bmatrix}}^{x} = \overbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}^{b}$$

unknown

Form of eqs. is $A\,x = b$

known

Use MATLAB to solve:

```
%Hanging Plate Problem 4.188
%Andy Ruina's solution, Jan 27, 2000

s=sqrt(2)/2;

A = [  1  0  0  0 -1  0
       0  1  0  0  0 -s
       0  0  1  1  0  s
       0  0 -1 -1  0  s
       0  0  1 -1  0 -s
       1 -1  0  0  1 -s ];

b = [0 0 1 0 0 0]';

x = A\b  %Solves simultaneous equations
```

MATLAB returns this

```
x =

    0.5000
    0.5000
    0.5000
    0.0000
    0.5000
    0.7071
```

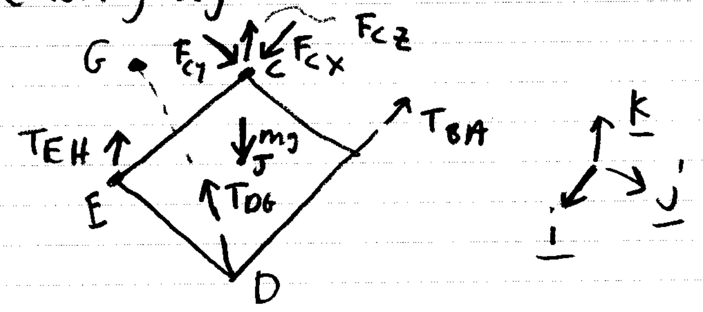
0.5000	F_{cx}/mg
0.5000	F_{cy}/mg
0.5000	F_{cz}/mg
0.0000	T_{EH}/mg
0.5000	T_{BA}/mg
0.7071	T_{DG}/mg

(f)

$$\Rightarrow \boxed{\begin{bmatrix} F_{cx} \\ F_{cy} \\ F_{cz} \\ T_{EH} \\ T_{BA} \\ T_{DG} \end{bmatrix} = mg \begin{bmatrix} .5 \\ .5 \\ .5 \\ 0 \\ .5 \\ \sqrt{2}/2 \end{bmatrix}}$$

(cont'd)

Reffering again to the FBD



g) $\underbrace{\left(\sum \underline{M}_{/D}\right)\cdot \underline{r}_{CD}}_{\text{moment about axis CD}} = 0$

T_{EH} is the only force which contributes. $\Rightarrow$ $\boxed{T_{EH} = 0}$

h) From part (b)

$\underbrace{\left(\sum \underline{M}_{/D}\right)\cdot \underline{r}_{DE}}_{\text{moment about axis DE}} = 0$

Only mg & F_{cz} contribute

$\Rightarrow$ $\boxed{F_{cz} = mg/2}$

$\underbrace{\left(\sum \underline{M}_{/C}\right)\cdot \underline{r}_{CG}}_{\text{moment about axis CG}} = 0$

Only mg & T_{BA} contribute

$$\left[\left(d T_{BA}\,\underline{k}\right) + \frac{d}{2} mg\,(\underline{j} - \underline{i})\right] \cdot (\underline{i} + \underline{k}) = 0$$

$$\Rightarrow T_{BA} + \frac{-mg}{2} = 0$$

$$\Rightarrow \boxed{T_{BA} = \frac{mg}{2}}$$

(cont'd)

$$\underbrace{\left(\Sigma \underline{M}_{/C}\right) \cdot \underline{i}}_{} = 0$$

Moment about axis CE

Only mg and T_{DG} contribute

$$\Rightarrow \left[(d\underline{i} + \underline{j}) \times [T_{DG}(-\underline{j} + \underline{k})/\sqrt{2}] + \frac{mgd}{2}(-\underline{i})\right] \cdot \underline{i} = 0$$

$$\Rightarrow \frac{T_{DG}}{\sqrt{2}} + \frac{-mg}{2} = 0 \Rightarrow \boxed{T_{DG} = \frac{\sqrt{2}}{2} mg}$$

$$\underbrace{\left(\Sigma \underline{M}_{/G}\right) \cdot \underline{i}}_{} = 0$$

Moment about $\underline{i}$ axis through G. Only mg & F_{cy} contribute. F_{cy} has twice the lever arm as mg, so $F_{cy} = \boxed{\frac{mg}{2}}$

So we could find 5 of the 6 unknowns one at a time, without knowing the others, by using moments about appropriate axes.

There is no simple eqn. (one eqn. in one unknown) to find F_{cx}.

Note, the 5 solns. above agree w/ the MATLAB solution.