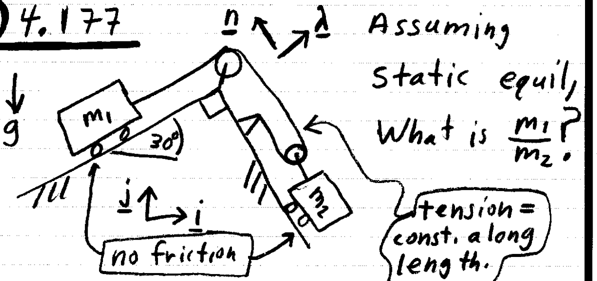
TAM 203, HW 1

due Feb 1, 2000

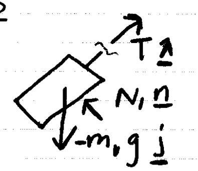
"Solutions" - A. Ruina

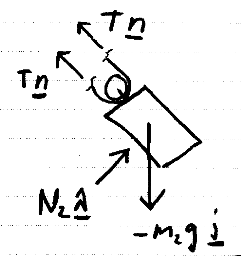
#1) 4.177

Assuming static equil, What is $\frac{m_1}{m_2}$?



FBDs





$\Sigma \underline{F} = \underline{0}$

$\Rightarrow \{-m_1 g \underline{j} + N_1 \underline{n} + T\underline{\lambda} = \underline{0}\}$

$\{\ \}\cdot\underline{\lambda} \Rightarrow$

$-m_1 g \underbrace{\underline{j}\cdot\underline{\lambda}}_{1/2} + N_1 \underbrace{\underline{n}\cdot\underline{\lambda}}_{0} + T \underbrace{\underline{\lambda}\cdot\underline{\lambda}}_{1} = \underbrace{\underline{0}\cdot\underline{\lambda}}_{0}$

$\Rightarrow \frac{1}{2} m_1 g = T \quad (1)$

$\Sigma \underline{F} = \underline{0}$

$\{-m_2 g \underline{j} + N_2 \underline{\lambda} + 2T\underline{n} = \underline{0}\}$

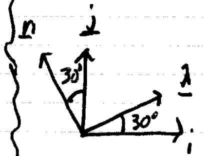
$\{\ \}\cdot\underline{n} \Rightarrow$

$-m_2 g \underbrace{\underline{j}\cdot\underline{n}}_{\sqrt{3}/2} + N_2 \underbrace{\underline{\lambda}\cdot\underline{n}}_{0} + 2T \underbrace{\underline{n}\cdot\underline{n}}_{1} = \underline{0}\cdot\underline{n}$

$\Rightarrow \frac{\sqrt{3}}{2} m_2 g = 2T \quad (2)$

$(1),(2) \Rightarrow \frac{m_1}{m_2} = \frac{2T/g}{4T/\sqrt{3}g} \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{\sqrt{3}}{2}}$

Geometry $\underline{i}, \underline{j}, \underline{\lambda}, \underline{n}$ are unit vectors



$\underline{i}\cdot\underline{i} = \underline{j}\cdot\underline{j} = \underline{\lambda}\cdot\underline{\lambda} = \underline{n}\cdot\underline{n} = 1$

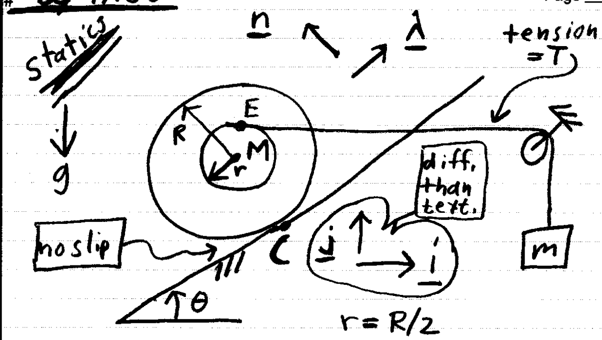
$\underline{i}\cdot\underline{j} = \underline{\lambda}\cdot\underline{n} = 0$

$\underline{j}\cdot\underline{\lambda} = \sin 30° = 1/2$

$\underline{j}\cdot\underline{n} = \cos 30° = \sqrt{3}/2$

2) 4.180

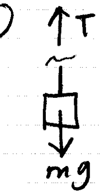
~~statics~~



Find m/M, T, reaction at C etc.

FBDs

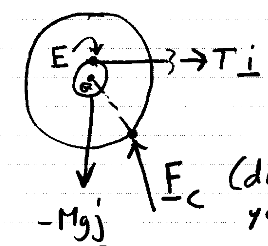
①



②

$\underline{r}_{CG} = R\underline{n}$

$\underline{r}_{CE} = R\underline{n} + r\underline{j}$



FOR FBD①: $\{\Sigma \underline{F} = \underline{0}\}\cdot\underline{j} \Rightarrow \boxed{T = mg}$

FOR FBD②:

$\sum \underline{M}_{/C} = \underline{0}$

$\Rightarrow \underline{r}_{CG} \times (-Mg\underline{j}) + \underline{r}_{CE} \times (T\underline{i}) = \underline{0}$

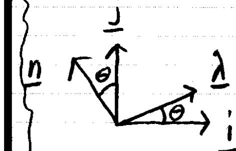
$(R\underline{n}) \times (-Mg\underline{j}) + (R\underline{n} + r\underline{j}) \times (T\underline{i}) = \underline{0}$

(1)

cont'd →

(2) (continued)

Geometry



$\underline{\lambda} = \cos\theta\, \underline{i} + \sin\theta\, \underline{j}$

$\underline{n} = -\sin\theta\, \underline{i} + \cos\theta\, \underline{j}$

$\underline{n} \times \underline{j} = -\sin\theta\, \underline{k}$

$\underline{n} \times \underline{i} = -\cos\theta\, \underline{k}$

$\underline{i} \times \underline{j} = \underline{\lambda} \times \underline{n} = \underline{k}$

etc.

$(1) \Rightarrow \{-RMg(-\sin\theta\, \underline{k}) + RT(-\cos\theta\, \underline{k}) + rT(-\underline{k}) = \underline{0}\}$

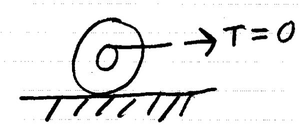
$\{\ \}\cdot\underline{k} \Rightarrow RMg\sin\theta - RT\cos\theta - rT = 0$

$\frac{r}{R} = \frac{1}{2}, T = mg \Rightarrow M\cancel{g}\sin\theta - m\cancel{g}(\cos\theta + \frac{1}{2}) = 0$

$\Rightarrow \boxed{\frac{m}{M} = \frac{\sin\theta}{\cos\theta + 1/2}}$ (a)

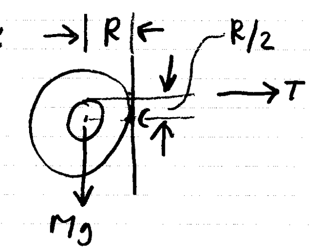
Checks:

$\theta = 0$:



formula gives 0 as it should

$\theta = \pi/2$:



tension has half the lever arm about pt. C $\Rightarrow$ $T = 2Mg \Rightarrow m = 2M$.

Formula above gives $\frac{m}{M} = 2$ as it should.

(cont'd)

#(2) (cont'd)

b) tension? $\boxed{T = mg = \frac{2Mg\sin\theta}{2\cos\theta+1}}$ (b)

(Note: no R in answer above.)

c) Reaction at C $\underline{F}_c = ?$

Go back to FBD ①

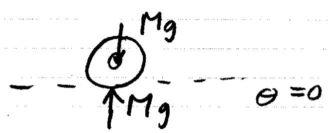
$\Sigma \underline{F}_i = \underline{0}$

$\Rightarrow -Mg\underline{j} + \underbrace{T}_{mg}\underline{i} + \underline{F}_c = \underline{0}$

$\Rightarrow \boxed{\underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}\underline{i} + \underline{j}\right]}$ (c)

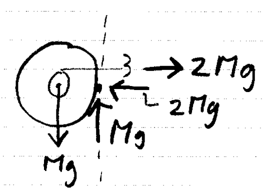
Checks

$\theta=0$



$\underline{F}_c = Mg\,\underline{j}$ (as per formula)

$\theta = \pi/2$

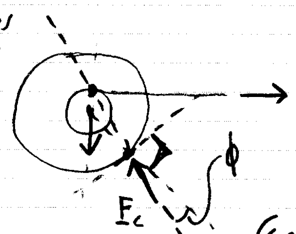


formula gives $\underline{F}_c = Mg(-2\underline{i} + \underline{j})$

as per FBD above

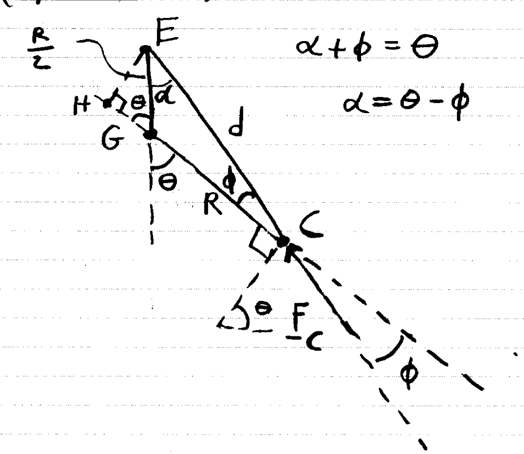
d) Use 3-force body logic

All 3 forces must have lines of action which intersect at E.



(cont'd)

#(2) (continued)



$\alpha + \phi = \theta$

$\alpha = \theta - \phi$

Look at triangle CEH:

$$\tan\phi = \frac{\frac{R}{2}\sin\theta}{R + \frac{R}{2}\cos\theta} = \frac{\sin\theta}{2+\cos\theta} \quad (*)$$

Now look at the soln. to part (c)

since $\underline{i} = \cos\theta\,\underline{\lambda} - \sin\theta\,\underline{n}$

and $\underline{j} = \sin\theta\,\underline{\lambda} + \cos\theta\,\underline{n}$

$$\Rightarrow \underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}(\cos\theta\,\underline{\lambda} - \sin\theta\,\underline{n}) + (\sin\theta\,\underline{\lambda} + \cos\theta\,\underline{n})\right]$$

$$= Mg\left[\frac{-2\sin\theta\cos\theta + (2\cos\theta+1)\sin\theta}{2\cos+1}\underline{\lambda} + \frac{+2\sin^2\theta + \cos\theta(2\cos\theta+1)}{2\cos\theta+1}\underline{n}\right]$$

$$= Mg\left[\frac{\sin\theta}{2\cos\theta+1}\underline{\lambda} + \frac{2+\cos\theta}{2\cos\theta+1}\underline{n}\right]$$

$$\Rightarrow \tan\phi = \frac{\left(\frac{\sin\theta}{2\cos\theta+1}\right)}{\left(\frac{2+\cos\theta}{2\cos\theta+1}\right)} = \frac{\sin\theta}{2+\cos\theta} \quad (**)$$

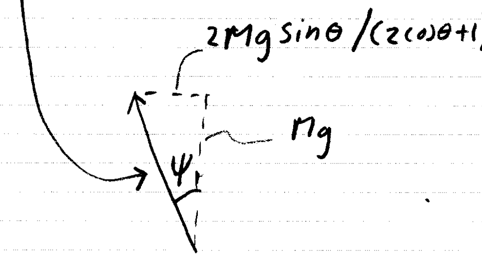
which checks w/ $*$ above

(cont'd)

#(2) (cont'd)

e) recall from (c)

$\underline{F}_c = Mg\left[\frac{-2\sin\theta}{2\cos\theta+1}\underline{i} + \underline{j}\right]$

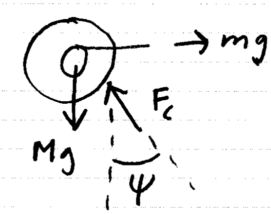


$\Rightarrow \tan\psi = \frac{2\sin\theta}{2\cos\theta+1}$

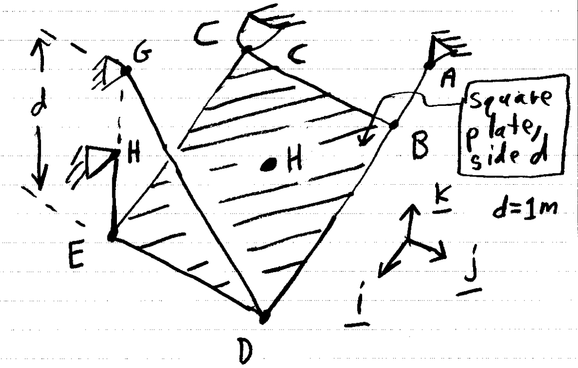
from part b: $\frac{m}{M} = \frac{2\sin\theta}{2\cos\theta+1}$

$\Rightarrow \boxed{\tan\psi = \frac{m}{M}}$ (e)

Note: despite all the algebra, this answer could be found directly from this FBD:

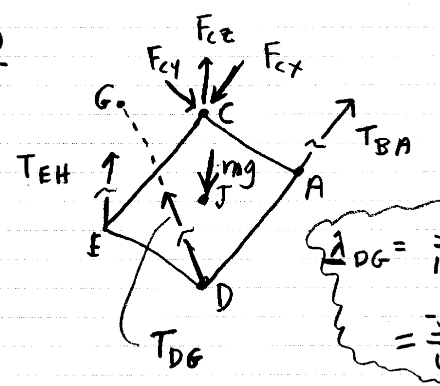


3) 4.188



Find the various tensions & reactions.

a) FBD



$$\underline{\lambda}_{DG} = \frac{\underline{r}_{DG}}{|\underline{r}_{DG}|} = \frac{-\underline{j}+\underline{k}}{\sqrt{2}}$$

b) Consider the axis DE. The only forces w/ nonzero moment are mg at H and F_{cz} at C. So $\boxed{F_{cz} = mg/2}$

c) $\sum \underline{F} = \underline{0}$

$$\Rightarrow F_{cx}\underline{i} + F_{cy}\underline{j} + F_{cz}\underline{k} + T_{EH}\underline{k} + T_{BA}(-\underline{i}) + T_{DG}\underline{\lambda}_{DG} - mg\underline{k} = \underline{0}$$

$$\underline{\lambda}_{DG} = \frac{-\underline{j}+\underline{k}}{\sqrt{2}}$$

(cont'd)

3) (cont'd)

d) $\sum \underline{M}_{/J} = \underline{0}$

$$\underline{r}_{C/J} \times \underline{F}_c + \underline{r}_{D/J} \times (T_{DG}\underline{\lambda}_{DG}) + \underline{r}_{E/J} \times (T_{EH}\underline{k}) + \underline{r}_{A/J} \times (T_{BA}(-\underline{i})) = \underline{0}$$

Factoring out a common factor of $d/2$ (.5m)

$$\Rightarrow (-\underline{i}-\underline{j}) \times (F_{cx}\underline{i} + F_{cy}\underline{j} + F_{cz}\underline{k}) + (\underline{i}-\underline{j}) \times (T_{EH}\underline{k}) + (\underline{i}+\underline{j}) \times T_{DG}\frac{(-\underline{j}+\underline{k})}{\sqrt{2}} + (-\underline{i}+\underline{j}) \times (T_{BA}(-\underline{i})) = \underline{0}$$

$$\Rightarrow [(\underline{k})F_{cx} + (-\underline{k})F_{cy} + (\underline{j}-\underline{i})F_{cz}] + [(-\underline{j}-\underline{i})T_{EH}] + [(\underline{i}-\underline{j}-\underline{k})\frac{T_{DG}}{\sqrt{2}}] + \underline{k}\,T_{BA} = \underline{0}$$

e) Write eqs. in comp. form.

$\sum \underline{F} = \underline{0} \Rightarrow$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}F_{cx} + \begin{bmatrix}0\\1\\0\end{bmatrix}F_{cy} + \begin{bmatrix}0\\0\\1\end{bmatrix}F_{cz} + \begin{bmatrix}0\\0\\1\end{bmatrix}T_{EH} + \begin{bmatrix}-1\\0\\0\end{bmatrix}T_{BA} + \begin{bmatrix}0\\-\sqrt{2}/2\\\sqrt{2}/2\end{bmatrix}T_{DG} = mg\begin{bmatrix}0\\0\\1\end{bmatrix}$$

1st comp, 2nd comp, 3rd comp make up 3 eqs in 6 unknowns.

(cont'd)

3) cont'd

$$\Rightarrow \begin{bmatrix}1 & 0 & 0 & 0 & -1 & 0\\0 & 1 & 0 & 0 & 0 & -\sqrt{2}/2\\0 & 0 & 1 & 1 & 0 & \sqrt{2}/2\end{bmatrix}\begin{bmatrix}F_{cx}\\F_{cy}\\F_{cz}\\T_{EH}\\T_{BA}\\T_{DG}\end{bmatrix} = mg\begin{bmatrix}0\\0\\1\end{bmatrix} \quad (*)$$

Likewise for moments ...

$\sum \underline{M}_{/J} = \underline{0}$

$$\Rightarrow \begin{bmatrix}0\\0\\1\end{bmatrix}F_{cx} + \begin{bmatrix}0\\0\\-1\end{bmatrix}F_{cy} + \begin{bmatrix}-1\\1\\0\end{bmatrix}F_{cz} + \begin{bmatrix}-1\\-1\\0\end{bmatrix}T_{EH} + \begin{bmatrix}0\\0\\1\end{bmatrix}T_{BA} + \begin{bmatrix}\sqrt{2}/2\\-\sqrt{2}/2\\-\sqrt{2}/2\end{bmatrix}T_{DG} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 0 & -1 & -1 & 0 & \sqrt{2}/2\\0 & 0 & 1 & -1 & 0 & -\sqrt{2}/2\\1 & -1 & 0 & 0 & 1 & -\sqrt{2}/2\end{bmatrix}\begin{bmatrix}F_{cx}\\F_{cy}\\F_{cz}\\T_{EH}\\T_{BA}\\T_{DG}\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad (**)$$

Putting (*) & (**) together and defining ^ variables as $\hat{(\ )} = \frac{(\ )}{mg}$ we get

(cont'd)

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\sqrt{2}/2 \\ 0 & 0 & 1 & 1 & 0 & \sqrt{2}/2 \\ 0 & 0 & -1 & -1 & 0 & \sqrt{2}/2 \\ 0 & 0 & 1 & -1 & 0 & -\sqrt{2}/2 \\ 1 & -1 & 0 & 0 & 1 & -\sqrt{2}/2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{F}_{Cx} \\ \hat{F}_{Cy} \\ \hat{F}_{Cz} \\ \hat{T}_{EH} \\ \hat{T}_{BA} \\ \hat{T}_{DG} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

Form of eqs. is $\underbrace{A}_{\text{known}} \underbrace{x}_{\text{unknown}} = \underbrace{b}_{\text{known}}$

Use MATLAB TO SOLVE:

```
%Hanging Plate Problem 4.188
%Andy Ruina's solution, Jan 27, 2000

s=sqrt(2)/2;

A = [  1  0  0  0 -1  0
       0  1  0  0  0 -s
       0  0  1  1  0  s
       0  0 -1 -1  0  s
       0  0  1 -1  0 -s
       1 -1  0  0  1 -s ];

b = [0 0 1 0 0 0]';

x = A\b  %Solves simultaneous equations
```

MATLAB returns this

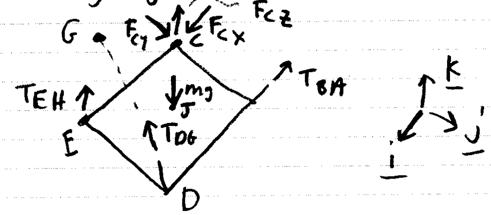
```
x =

    0.5000   Fcx/mg
    0.5000   Fcy/mg
    0.5000   Fcz/mg
    0.0000   TEH/mg
    0.5000   TBA/mg
    0.7071   TDG/mg
```

(f)

$$\Rightarrow \begin{bmatrix} F_{Cx} \\ F_{Cy} \\ F_{Cz} \\ T_{EH} \\ T_{BA} \\ T_{DG} \end{bmatrix} = mg \begin{bmatrix} .5 \\ .5 \\ .5 \\ 0 \\ .5 \\ \sqrt{2}/2 \end{bmatrix}$$

(cont'd)

Reffering again to the FBD



g) $\underbrace{(\Sigma \underline{M}_{/D}) \cdot \underline{r}_{CD}}_{\text{moment about axis CD}} = 0$

T_{EH} is the only force which contributes. $\Rightarrow$ $\boxed{T_{EH} = 0}$

h) From part (b)

$\underbrace{(\Sigma \underline{M}_{/D}) \cdot \underline{r}_{DE}}_{\text{moment about axis DE}} = 0$

Only mg & F_{Cz} contribute

$\Rightarrow$ $\boxed{F_{Cz} = mg/2}$

$\underbrace{(\Sigma \underline{M}_{/C}) \cdot \underline{r}_{CG}}_{\text{moment about axis CG}} = 0$

Only mg & T_{BA} contribute

$$\left[(d T_{BA} \underline{k}) + \frac{d}{2} mg (\underline{j} - \underline{i})\right] (\underline{i} + \underline{k}) = 0$$

$$\Rightarrow T_{BA} + \frac{-mg}{2} = 0$$

$$\Rightarrow \boxed{T_{BA} = \frac{mg}{2}}$$

(cont'd)

$\underbrace{(\Sigma \underline{M}_{/C}) \cdot \underline{i}}_{\text{Moment about axis CE}} = 0$

Only mg and T_{DG} contribute

$$\Rightarrow \left[(d\underline{i} + \underline{j}) \times [T_{DG} (-\underline{j} + \underline{k})/\sqrt{2}] + \frac{mgd}{2}(-\underline{i})\right] \cdot \underline{i} = 0$$

$$\Rightarrow \frac{T_{DG}}{\sqrt{2}} + \frac{-mg}{2} = 0 \Rightarrow \boxed{T_{DG} = \frac{\sqrt{2}}{2} mg}$$

$(\Sigma \underline{M}_{/G}) \cdot \underline{i} = 0$

Moment about $\underline{i}$ axis through G. Only mg & F_{Cy} contribute. F_{Cy} has twice the lever arm as mg, so $F_{Cy} = \boxed{\frac{mg}{2}}$

So we could find 5 of the 6 unknowns one at a time, without knowning the others, by using moments about appropriate axes.

There is no simple eqn. (one eqn. in one unknown) to find F_{Cx}.

Note, the 5 solns. above agree w/ the MATLAB solution.