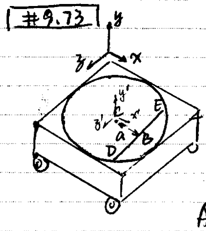
HW#13 Dynamics 203 — 2000

#9.73



A turntable oscillates with displacement $\vec{x}_c(t) = A\sin(st)\hat{i}$

A bug walks along DE with v_0 relative to the turntable.

At the moment, $x = A$, DE is parallel to z axis. $|CB| = a$. $\omega_0, \dot{\omega}_0$ given. find $\vec{a}_b$

We consider a frame $\mathcal{B}$ attached to the turntable with its origin at C.

$\vec{a}_b = \vec{a}_{b'} + \vec{a}_{col} + \vec{a}_{rel}$

where $\vec{a}_{b'}$ = acceleration of a point B' that is fixed in $\mathcal{B}$ and at the moment coincides with B.

$\vec{a}_{col}$ = Coriolis acceleration

$\vec{a}_{rel}$ = Acceleration of B relative to frame $\mathcal{B}$

$\vec{a}_{b'} = \vec{a}_c + \dot{\vec{\omega}}_{\mathcal{B}} \times \vec{r}_{B/C} + \vec{\omega}_{\mathcal{B}} \times (\vec{\omega}_{\mathcal{B}} \times \vec{r}_{B/C})$

$= -As^2\sin(st)\hat{i} + \dot{\omega}_0\hat{j} \times a\hat{i} + \omega_0\hat{j} \times (\omega_0\hat{j} \times a\hat{i})$

$= -(As^2 + a\omega_0^2)\hat{i} - a\dot{\omega}_0\hat{k}$ (since at this instant $A = A\sin(st)$. $\therefore \sin(st) = 1$)

$\vec{a}_{col} = 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{rel}$

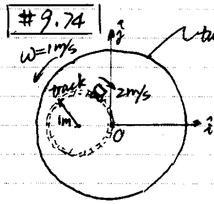
$= 2\omega_0\hat{j} \times (-v_0)\hat{k}$

$= -2\omega_0 v_0\hat{i}$

$\vec{a}_{rel} = 0$

$\therefore \vec{a}_b = -(As^2 + a\omega_0^2 + 2\omega_0 v_0)\hat{i} - a\dot{\omega}_0\hat{k}$ m/s²

#9.74



0.1 ~~kg~~ toy train is going clockwise at a constant rate.

turntable is rotating counterclockwise.

At the instant, train is pointing due south $(-\hat{j})$ and is at the center O of the turntable.

Consider the frame $\mathcal{B}_{(x\text{-}y)}$ attached to the turntable, origin at

a) velocity of the train relative to the turntable?

$\vec{v}_{\mathcal{B}} = -2\hat{j}$ m/s

#9.74 Cont'd

b) What's the absolute velocity?

$\vec{v} = \vec{v}_{B'} + \vec{v}_{rel}$ B' is the point coinciding with the train but fixed on the rotating frame $\mathcal{B}$, at the instant.

$\vec{v}_{B'} = \cancel{\vec{v}_0} + \vec{\omega}_{\mathcal{B}} \times \cancel{\vec{r}_{B/O}} = 0$ $\vec{v}_{rel} = -2\hat{j}$ m/s

$\therefore \vec{v} = -2\hat{j}$ m/s

c) Acceleration relative to the turntable?

$\vec{a}_{\mathcal{B}} = -\frac{v^2}{r}\hat{i} = -4\hat{i}$ m/s²

d) Absolute acceleration?

$\vec{a} = \vec{a}_{B'} + \vec{a}_{col} + \vec{a}_{rel}$ B' is the point coinciding with the train but fixed on the rotating frame $\mathcal{B}$ at the instant

where $\vec{a}_{B'} = \cancel{\vec{a}_0} + \cancel{\dot{\vec{\omega}}_{\mathcal{B}}} \times \cancel{\vec{r}_{B/O}} + \vec{\omega}_{\mathcal{B}} \times (\vec{\omega}_{\mathcal{B}} \times \cancel{\vec{r}_{B/O}}) = 0$

$\vec{a}_{col} = 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{rel} = 2(1\hat{k}) \times (-2\hat{j}) = 4\hat{i}$ m/s²

$\vec{a}_{rel} = -4\hat{i}$ m/s²

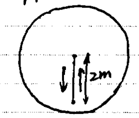
$\therefore \vec{a} = 0$ m/s²

e) Total force acting on train?

$\Sigma\vec{F} = m\vec{a} = 0$ $\therefore$ The total force is 0.

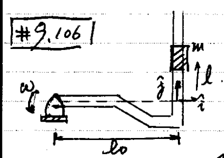
f) Sketch.

Suppose $t=0$, the train is at the center.



goes back & forth along the vertical line.

#9.106

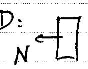


No friction, bead starts at $l=0$, $\dot{l} = v_0$, $\omega = \omega_0$ constant.

Find $l(t)$

Set up the frame $\mathcal{B}(x\text{-}y)$ attached the instantaneously vertical body rotating with

FBD:



$\vec{a} = \vec{a}_{\mathcal{B}} + \vec{a}_{col} + \vec{a}_{rel}$

$\vec{a}_{\mathcal{B}} = \vec{a}_O + \overset{0}{\cancel{\dot{\vec{\omega}}}} \times \vec{r}_{m/O} + \vec{\omega}_{\mathcal{B}} \times (\vec{\omega}_{\mathcal{B}} \times \vec{r}_{m/O})$

$= -l_0\omega_0^2\hat{i} - l\omega_0^2\hat{j}$

$\vec{a}_{col} = 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{rel} = -2\dot{l}\omega_0\hat{i}$

$\vec{a}_{rel} = \ddot{l}\hat{j}$

$\therefore \vec{a} = (-l_0\omega_0^2 - 2\dot{l}\omega_0)\hat{i} + (-l\omega_0^2 + \ddot{l})\hat{j}$

$\Sigma\vec{F} = m\vec{a}$

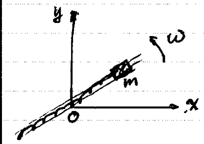
$\{\ \}\cdot\hat{j}$: $\ddot{l} - l\omega_0^2 = 0$

$\therefore l = A_1 e^{\omega_0 t} + A_2 e^{-\omega_0 t}$

$t=0$, $l=0$, $\dot{l} = v_0$ $\therefore \begin{cases} A_1 + A_2 = 0 \\ A_1\omega_0 - A_2\omega_0 = v_0 \end{cases}$ $\therefore \begin{cases} A_1 = \frac{v_0}{2\omega_0} \\ A_2 = -\frac{v_0}{2\omega_0} \end{cases}$

$\therefore l(t) = \frac{v_0}{2\omega_0}(e^{\omega_0 t} - e^{-\omega_0 t})$

#9.113



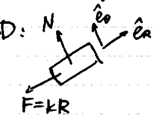
No friction. ω constant, spring constant k.

Spring is relaxed when bead is at the center position.

Assume $t=0$, $R=d$, $\dot{R}=0$. If needed, may assume $k > m\omega^2$

a) Derive eqn for the position of the bead R(t).

FBD:



(neglect gravity)

$\Sigma\vec{F} = m\vec{a}$

$\therefore -kR\hat{e}_R + N\hat{e}_\theta = m[(\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta]$

$\{\ \}\cdot\hat{e}_R$: $-kR = m\ddot{R} - mR\dot{\theta}^2$ (*)

$\{\ \}\cdot\hat{e}_\theta$: $N = 2m\dot{R}\dot{\theta}$

From (*): $\ddot{R} + \frac{k - m\omega^2}{m}R = 0$ (assume $k > m\omega^2$)

$\therefore R = A_1\sin\sqrt{\frac{k-m\omega^2}{m}}\,t + A_2\cos\sqrt{\frac{k-m\omega^2}{m}}\,t$

since $t=0$, $R=d$, $\dot{R}=0$ $\therefore A_1 = 0$, $A_2 = d$.

$\therefore R = d\cos\sqrt{\frac{k-m\omega^2}{m}}\,t$

b) How is the motion affected by large k vs small k?

from $R = d\cos\sqrt{\frac{k-m\omega^2}{m}}\,t$, we can see that when k is very large, $\sqrt{\frac{k-m\omega^2}{m}}$ is large. (when $k \gg m\omega^2$, $\sqrt{\frac{k-m\omega^2}{m}} \approx$ const) the frequency of the motion is also large.

When k is small, frequency decreases.

#9.113 Cont'd

c) When bead passes through the center position, $R=0$

$\therefore \cos\sqrt{\frac{k-m\omega^2}{m}}t = 0$. $\sin\sqrt{\frac{k-m\omega^2}{m}} = 1$

$\therefore N = 2m\dot{R}\dot{\theta} = 2md\sqrt{\frac{k-m\omega^2}{m}}(-\sin\sqrt{\frac{k-m\omega^2}{m}}t)\omega$

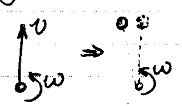
$\therefore |N| = 2md\omega\sqrt{\frac{k-m\omega^2}{m}}$

the magnitude of the force acting on the bead.

d) Corriolis acceleration:

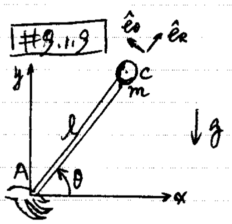
$$2\dot{R}\dot{\theta}\hat{e}_\theta = -2md\omega\sqrt{\frac{k-m\omega^2}{m}}\sin\sqrt{\frac{k-m\omega^2}{m}}t\ \hat{e}_\theta$$

Example: you throw a particle and find it falls to ground a distance east of the point supposed to be.



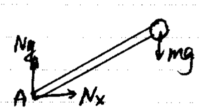
Due to the rotation of the earth. $\vec{a} = 2\vec{\omega}\times\vec{v}_{rel}$. Coriolis accleration, eastward.

#9.119



rod on palm A, length l, mass m at C
Assume $\dot{\theta}$ & θ are known at the instant of interest. hand is accelerating
$\vec{a}_{hand} = a_{hx}\hat{i} + a_{hy}\hat{j}$

a) Draw FBD



b) Assume hand is stationary. Solve for $\ddot{\theta}$

From the acceleration in polar coordinates:

$(\ddot{R}-R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta}+R\ddot{\theta})\hat{e}_\theta$

$R=l \therefore \ddot{R}=\dot{R}=0$ $\quad\therefore \vec{a}_c = -R\dot{\theta}^2\hat{e}_R + R\ddot{\theta}\hat{e}_\theta$

$\hat{e}_R = \cos\theta\hat{i} + \sin\theta\hat{j}$ $\quad \hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$

$\therefore \dot{\vec{H}}_{/A} = \vec{r}_{/A}\times m_c\vec{a}_c$

$= l(\cos\theta\hat{i}+\sin\theta\hat{j})\times m[-l\dot{\theta}^2(\cos\theta\hat{i}+\sin\theta\hat{j}) + l\ddot{\theta}(-\sin\theta\hat{i}+\cos\theta\hat{j})]$ in cartesian

$= ml^2\ddot{\theta}\hat{k}$ in polar coordinates

$\Sigma\vec{M}_{/A} = \vec{r}_{/A}\times(-mg\hat{j}) = -mgl\cos\theta\hat{k}$

$\therefore \{\ \}\cdot\hat{k}:\ ml^2\ddot{\theta} = -mgl\cos\theta$

$\therefore \ddot{\theta} = -\frac{g\cos\theta}{l}$

#9.119 cont.

c) If hand is not stationary, but $\ddot{\theta}$ is given.

$\Sigma\vec{F} = m\vec{a}$. where $\vec{a} = \vec{a}_{hand} + \vec{a}_{c/hand}$

$\{\ \}\cdot\hat{j}:\ N_y - mg = ma_{cy} + ma_{hy}$

And $a_{cy} = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta$.

$\therefore N_y = mg + ma_{hy} + m(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)$