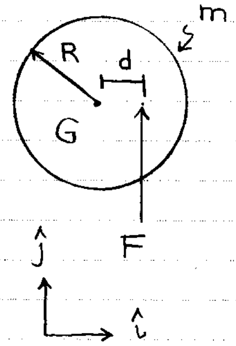
HW 12 Solutions by David Cabrera (4/24/00)

9.25 A thin flat disk is floating in space. A force F is applied at a distance d from the center in the y-direction.

FBD



a) What is $\underline{a}_G$?

LMB: $\Sigma \underline{F} = m\underline{a}_G$

$$F\hat{j} = m\underline{a}_G$$

$$\Rightarrow \boxed{\underline{a}_G = \frac{F}{m}\hat{j}}$$

b) What is $\underline{\alpha}$?

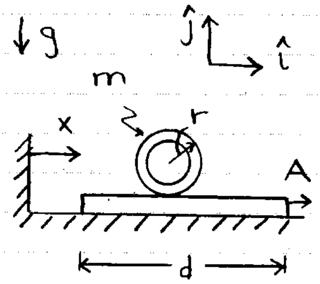
AMB/G: $\Sigma \underline{M}_{/G} = \dot{\underline{H}}_{/G}$

$$d\hat{\imath} \times F\hat{j} = I_{zz}^G \alpha \hat{k}$$ where $I_{zz}^G = \frac{1}{2}mR^2$ from Table IV

$$\{Fd\hat{k} = \tfrac{1}{2}mR^2\alpha\hat{k}\}$$

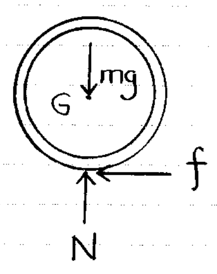
$$\{\}\cdot\hat{k} \Rightarrow \quad \alpha = \frac{2Fd}{mR^2} \Rightarrow \boxed{\underline{\alpha} = \frac{2Fd}{mR^2}\hat{k}}$$

9.37 A napkin ring lies on a tablecloth and rolls w/o slip as a child pulls the tablecloth w/ accel. A. The ring starts @ x = d.



a) What is $\underline{a}_G$?

FBD of ring



LMB: $\Sigma \underline{F} = m\underline{a}_G$

$$-f\hat{\imath} + (N - mg)\hat{j} = m\underline{a}_G$$

where $\underline{a}_G = a_G\hat{\imath}$ assuming the ring stays on the cloth

LMB $\cdot\hat{\imath} \Rightarrow \quad -f = ma_G \quad (1)$

AMB/G: $\{\Sigma \underline{M}_{/G} = \dot{\underline{H}}_{/G}\}\cdot\hat{k}$

$$\Rightarrow \quad -fr = I\alpha \quad (2)$$

where $I = \underbrace{I_{zz}^G = mR^2}_{\text{from Table IV}}$

(continued)

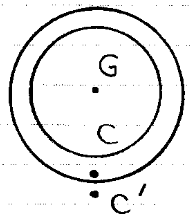
(1) & (2) give $a_G = r\alpha$, $\frac{dv_G}{dt} = r\frac{d\omega}{dt}$ (3)

Integrating and using $v_G = 0, \omega = 0$ @ $t = 0$:

$v_G = r\omega$ (4) ← true for all time whether or not the ring is rolling or is on the tablecloth

No slip $\Rightarrow \underline{a}_c \cdot \hat{\imath} = \underline{a}_{c'} \cdot \hat{\imath} = A$

(i.e. the tangential accelerations match)



To find $\underline{a}_G$:

$\underline{a}_G = \underline{a}_c + \underline{a}_{G/c}$

where $\underline{a}_{G/c} = -\alpha r\hat{\imath}$ since the ring rolls on the cloth

C: pt. on ring
C': pt. on cloth
C & C' are coincident

$\Rightarrow \{a_G\hat{\imath} = \underline{a}_c - \alpha r\hat{\imath}\}\cdot\hat{\imath}$

$\Rightarrow a_G = \underline{a}_c \cdot \hat{\imath} - \alpha r$
$= A - \alpha r$ from no slip condition above

$\therefore a_G = A - \alpha r$ (5)

(3) & (5) give $a_G = \frac{A}{2} \Rightarrow \boxed{\underline{a}_G = \frac{A}{2}\hat{\imath}}$

b) When the ring rolls off the left hand end, it has moved a distance d along the tablecloth. Need to look @ motion with respect to the tablecloth:

$\underline{a}_{G/c'} = \underline{a}_G - \underline{a}_{c'}$
$= \frac{A}{2}\hat{\imath} - A\hat{\imath} = -\frac{A}{2}\hat{\imath}$

$\Rightarrow \underline{v}_{G/c'} = -\frac{A}{2}t\,\hat{\imath}$, $\underline{x}_{G/c} = -\frac{A}{4}t^2\,\hat{\imath}$

The ring rolls off when $|\underline{x}_{G/c}| = d$

$\Rightarrow d = \frac{A}{4}t^{*2} \Rightarrow t^* = 2\sqrt{d/A}$

Since the tablecloth has acceleration $a_{c'} = |\underline{a}_{c'}| = A \Rightarrow s = \frac{1}{2}At^2$

(continued)

where s = displacement of any pt. on the tablecloth

So, when $t = t^* = 2\sqrt{d/A}$,

$$s = \tfrac{1}{2}A\left(\tfrac{4d}{A}\right) \Rightarrow \boxed{s = 2d}$$

c) When the ring hits the ground, look at the velocity of the contact pt.:

$$\underline{v}_c = \underline{v}_G + \underline{\omega} \times \underline{r}_{c/G}$$

where $\underline{v}_G = \frac{A}{2} t^* \hat{\imath} = \sqrt{Ad}\,\hat{\imath}$ (since $\underline{a}_G = \frac{A}{2}\hat{\imath}$)

and $\underline{\omega} = \frac{A}{2r} t^* \hat{k} = \frac{1}{r}\sqrt{Ad}\,\hat{k}$ (since $\underline{\alpha} = \frac{A}{2r}\hat{k}$)

$$\Rightarrow \underline{v}_c = \sqrt{Ad}\,\hat{\imath} + \tfrac{1}{r}\sqrt{Ad}\,\hat{k} \times -r\hat{\jmath}$$
$$= 2\sqrt{Ad}\,\hat{\imath}$$

Since $\underline{v}_c \neq \underline{0}$, the ring does not roll at first. It will slide until $\underline{v}_c = \underline{0}$ or $v_G = -\omega r$. But, since (4) holds for all time, we have

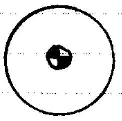
$$v_G = \omega r,\ v_G = -\omega r \Rightarrow \boxed{\begin{array}{c} v_G = 0 \\ \omega = 0 \end{array}}$$

So, the ring will eventually stop after sliding for some time after it leaves the tablecloth.

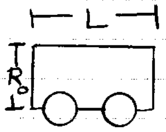
d) Even if the ring slips while it is on the tablecloth, (4) still holds. Also, the transition condition from slipping to rolling still holds ($\underline{v}_c = \underline{0}$ OR $v_G = -\omega r$).

$\therefore$ $\boxed{v_G = 0,\ \omega = 0}$ just like above

Which object wins the race down the slip resistant 30° slope?

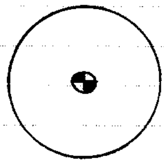
ref: 

disk
$m = M_o$
$r = R_o$

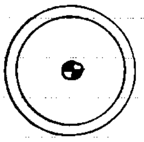
A) 

$m = M_o$

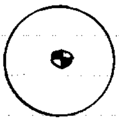
block (massless wheels, frictionless bearings)

B) 

disk
$m = M_o$,
$r = 2R_o$

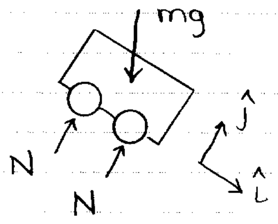
C) 

hollow pipe
$m = M_o$
$r = R_o$

D) 

disk
$m = 2M_o$
$r = R_o$

FBD (block)

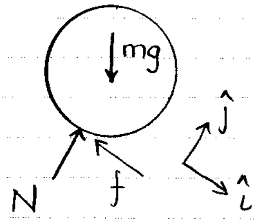


LMB: $\{\Sigma \underline{F} = m\underline{a}\} \cdot \hat{i}$

$\Rightarrow mg\sin\theta = ma$

$\therefore \boxed{a = g\sin\theta}$

FBD (round object)



LMB: $\{\Sigma \underline{F} = m\underline{a}\}$

$\{\}\cdot\hat{i} \Rightarrow mg\sin\theta - f = ma$ (1)

AMB/cm: $\{\Sigma \underline{M}_{/cm} = \dot{\underline{H}}_{/cm}\} \cdot \hat{k}$

$\Rightarrow -fr = -I\alpha$ where $\underline{\alpha} = -\alpha\hat{k}$, $I = I_{zz}^{cm}$

$\therefore f = \frac{I\alpha}{r}$ (2)

Rolling condition: $a = r\alpha$ (3)

(3) $\Rightarrow \alpha = \frac{a}{r}$

(2) $\Rightarrow f = \frac{I}{r}\left(\frac{a}{r}\right) = \frac{Ia}{r^2}$

Plugging the above expressions into (1):

$mg\sin\theta - \frac{Ia}{r^2} = ma$

$\Rightarrow \boxed{a = g\sin\theta\left(\frac{1}{1 + \frac{I}{mr^2}}\right)}$

(continued)

Since the accelerations are constant and the objects start at the same pt. the object with the largest accel. will win.

Disk: $I = \frac{1}{2}mr^2 \Rightarrow a = g\sin\Theta\left(\frac{1}{1+\frac{\frac{1}{2}mr^2}{mr^2}}\right)$

$\therefore a = \frac{2}{3}g\sin\Theta$ for any disk

Hollow pipe: $I = mr^2 \Rightarrow a = g\sin\Theta\left(\frac{1}{1+\frac{mr^2}{mr^2}}\right)$

$\therefore a = \frac{1}{2}g\sin\Theta$ for any pipe

Order of finish:

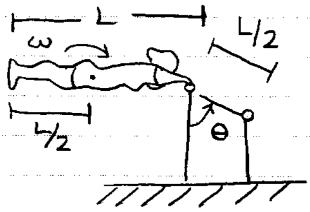
1^{ST}: Block (A)

(tie) 2^{ND}: All disks (ref, B, D)

5^{th}: Pipe (C)

- A massless cylinder with a point mass at its center has $I = 0$. So, it will accelerate as fast the block.
- To see an object which goes slower than the pipe, see Sample 9.14 in the book. (a spool w/ the inner radius in contact w/ the slope).
- Any round object with inertia will be slower than the block (the kinetic energy has both a translational and a rotational part), so the block is the fastest

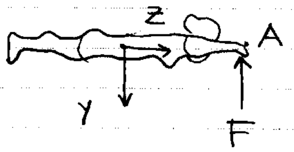
A gymnast performs the uneven parallel bars w/ radii of gyration k_x, k_y, k_z about her center of mass. Just before she grasps the top bar, her body is horizontal and rotating w/ angular rate ω w/ her CM stationary.



a) What is the gymnast's rotation rate just after she grasps the bar?

approximation: the reaction force F is much greater than mg
$\Rightarrow$ neglect mg

FBD (of gymnast @ collision)



AMB/A : $\underbrace{\Sigma \underline{M}_{/A}}_{0} = \dot{\underline{H}}_{/A}$

$\Rightarrow \underline{H}_{/A}$ conserved

① just before collision
② just after collision
③ just before her hips hit the lower bar

$$\underline{H}_{/A}^{①} = \underline{r}_{CM/A} \times m\cancel{\underline{v}_{CM}}^{0} + I_{xx}\,\omega\,\hat{\imath}$$

$$\Rightarrow \underline{H}_{/A}^{①} = m k_x^2 \omega\, \hat{\imath}$$

$$\underline{H}_{/A}^{②} = I_{xx}^{A}\, \omega_2 \hat{\imath} \quad \text{(A is a fixed pt.)}$$
$$= \left(mk_x^2 + m\left(\tfrac{L}{2}\right)^2\right)\omega_2 \hat{\imath}$$
$$= m\left(k_x^2 + \tfrac{1}{4}L^2\right)\omega_2 \hat{\imath}$$

$$\left\{ \underline{H}_{/A}^{②} = \underline{H}_{/A}^{①} \right\} \cdot \hat{\imath}$$

$$\Rightarrow \boxed{\omega_2 = \frac{k_x^2}{k_x^2 + \frac{L^2}{4}}\,\omega}$$

b) What is $|\underline{v}_{CM}|$ as her hips strike the lower bar?

As the gymnast rotates about the top bar, energy is conserved.

$$E_{k_2} + E_{P_2} = E_{k_3} + E_{P_3}$$

let $E_P = 0$ at upper bar height

(continued)

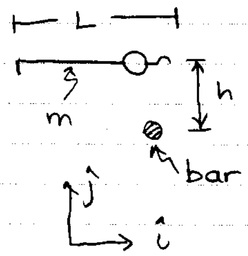
$$\Rightarrow \underbrace{\tfrac{1}{2} I_{xx}^{A}\, \omega_2^2}_{\text{since A is fixed}} + 0 = \underbrace{\tfrac{1}{2} I_{xx}^{A}\, \omega_3^2}_{\text{since A is fixed}} - mg\left(\tfrac{L}{2}\cos\Theta\right)$$

$$\tfrac{1}{\cancel{3}}\cancel{m}\left(k_x^2+\tfrac{L^2}{4}\right)\left[\frac{k_x^2}{k_x^2+\frac{L^2}{4}}\,\omega\right]^2 = \tfrac{1}{\cancel{3}}\cancel{m}\left(k_x^2+\tfrac{L^2}{4}\right)\omega_3^2 - \tfrac{1}{\cancel{2}}\cancel{m}gL\cos\Theta$$

$$\boxed{\begin{aligned} &\Rightarrow \omega_3 = \left[\left(\frac{k_x^2}{k_x^2+L^2/4}\right)^2\omega^2 + \frac{gL\cos\Theta}{k_x^2+L^2/4}\right]^{1/2} \\ &\Rightarrow |\underline{v}_{CM}| = \omega_3\left(\tfrac{L}{2}\right) \end{aligned}}$$

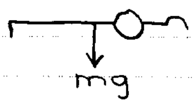
c) When the gymnast's hips hit the lower bar (@ her CM) and her hands release from the upper bar, she will rotate about the lower bar at the rate ω_3 (conservation of angular momentum about her CM).

9.59 An acrobat modeled as a uniform rigid mass m of length ℓ falls w/o rotation from height h where she was stationary. She then grabs the bar w/ a firm but slippery grip.



What is h s.t. she ends up in a stationary handstand?

FBD (①→②)



① shown position
② just before collision
③ just after collision
④ handstand position

Energy conserved

$$\Rightarrow E_{K_1} + E_{P_1} = E_{K_2} + E_{P_2}$$

$$0 + mgh = \tfrac{1}{2} m v_{CM}^2 + 0$$

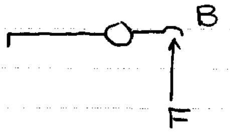
$$\Rightarrow v_{CM} = \sqrt{2gh}$$

($E_P = 0$ at bar height)

(continued)

$\underline{AMB_{/B}}$: $\underbrace{\Sigma \underline{M}_{/B}}_{0} = \dot{\underline{H}}_{/B}$

FBD (② → ③)



(F huge compared to mg)

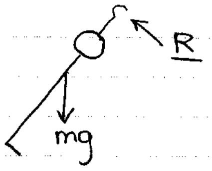
$\Rightarrow$ $\underline{H}_{/B}$ conserved

$\underline{H}_{/B}^{②} = \underline{r}_{CM/B} \times m\underline{v}_{CM}$
$= -\frac{L}{2}\hat{\imath} \times m\sqrt{2gh}\,(-\hat{\jmath})$
$= \frac{1}{2} mL\sqrt{2gh}\,\hat{k}$

$\underline{H}_{/B}^{③} = I_{zz}^{B}\,\omega\,\hat{k}$ ← since B is fixed
$= \frac{1}{3} mL^2 \omega\,\hat{k}$

$\{\underline{H}_{/B}^{③} = \underline{H}_{/B}^{②}\}\cdot\hat{k} \Rightarrow \omega = \dfrac{3\sqrt{2gh}}{2L}$

FBD (③ → ④)



For the motion from ③ to ④ the reaction force $\underline{R}$ does no work
$\Rightarrow$ energy conserved

$E_{K_3} + E_{P_3} = E_{K_4} + E_{P_4}$

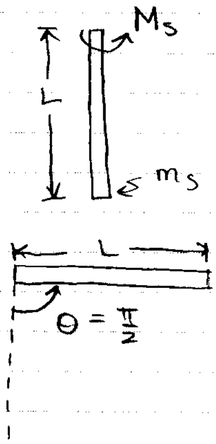
$\frac{1}{2} I_{zz}^{B} \omega_3^2 + 0 = 0 + mg\left(\frac{L}{2}\right)$

$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\left(\frac{9(2gh)}{4L^2}\right) = \frac{1}{2}mgL$

$\Rightarrow$ $\boxed{h = \frac{2}{3}L}$

<u>9.61</u> A baseball bat is approximated as a rigid stick w/ mass m_s and length L. The swinging occurs by application of a constant torque M_s at the hands over an angle $\theta = \pi/2$. A ball arrives at a length l along the bat at some speed v_b. There is an elastic collision.

FBD



(continued)

a) To find ω just before collision, use energy balance:

$$M_s \Delta\Theta = \frac{1}{2} I_{zz}^{H} \omega^2$$

$$\Rightarrow M_s\left(\frac{\pi}{2}\right) = \frac{1}{6} m L^2 \omega^2$$

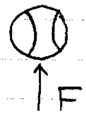
$$\therefore \boxed{\omega = \frac{1}{L}\sqrt{\frac{3\pi M_s}{m_s}}}$$

The total energy just before the collision is simply

$$\boxed{E = M_s\left(\frac{\pi}{2}\right) + \frac{1}{2} m_b v_b^2}$$

For the ball:

FBD of ball during collision

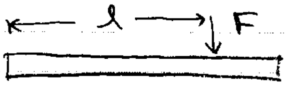


LMB: $\int_{t_1}^{t_2} \underline{F} dt = m_b \underline{v}_{HIT} - m_b \underline{v}_b$

$$\left\{\int_{t_1}^{t_2} F\hat{j}\, dt = m_b v_{HIT}\hat{j} + m_b v_b \hat{j}\right\}\cdot\hat{j}$$

$$\Rightarrow \boxed{\int F dt = m_b (v_{HIT} + v_b)} \quad (1)$$

FBD of bat during collision



For the bat:

AMB/H: $\int_{t_1}^{t_2} \Sigma \underline{M}_{/H}\, dt = \underline{H}_{/H}^{2} - \underline{H}_{/H}^{1}$

$$\Rightarrow \int_{t_1}^{t_2} l\hat{i} \times F(-\hat{j})\, dt = I_{zz}^{H} \omega_2 \hat{k} - I_{zz}^{H} \omega \hat{k}$$

$$\left\{-l\int_{t_1}^{t_2} F dt\ \hat{k} = \frac{1}{3} m L^2 (\omega_2 - \omega)\hat{k}\right\}\cdot\hat{k}$$

$$\Rightarrow \boxed{\int F dt = \frac{mL^2}{3l}(\omega - \omega_2)} \quad (2)$$

Want to find the $\int F dt$ that conserves energy:

$$(E_{K_1})_{BAT} + (E_{K_1})_{BALL} = (E_{K_2})_{BAT} + (E_{K_2})_{BALL}$$

$$\boxed{\begin{aligned}\frac{\pi}{2} M_s + \frac{1}{2} m_b v_b^2 &= \frac{1}{2} I_{zz}^{H} \omega_2^2 + \frac{1}{2} m_b v_{HIT}^2 \\ &= \frac{1}{6} m_s L^2 \omega_2^2 + \frac{1}{2} m_b v_{HIT}^2\end{aligned}} \quad (3)$$

Now we have 3 equations (1), (2), (3) in 3 unknowns ($\int F dt$, ω_2, v_{HIT}).

Note: $\dfrac{\text{separation speed}}{\text{approach speed}} = \dfrac{v_{HIT} - \omega_2 l}{v_b + \omega l} \leftarrow$ should be 1

(continued)

Solving for v_{HIT} in closed form is quite tedious. You should plug in numbers to find v_{HIT}:

Let $m_b = 8$ oz., $L = 32$ in., $v_b = 90$ mph.

To get a value of M_s, note that Mark McGwire's bat speed (tip of bat) is about 100 mph. Take half of that and use

$$\omega L = \tfrac{1}{2}(100\text{ mph}) = \sqrt{\frac{3\pi M_s}{m_s}} \Bigg\} \text{from previous page}$$

where $m_s = 30$ oz, say.

$$\Rightarrow \quad M_s \approx 33.2 \text{ lb}_f \cdot \text{ft}$$

We'd like to vary m_s and ℓ to see how v_{HIT} changes. So, we'd like to get one equation involving v_{HIT}, m_s, ℓ, and known quantities.

$$(1) \,\&\, (2) \Rightarrow m_b(v_{HIT} + v_b) = \frac{m_s L^2}{3\ell}(\omega - \omega_2)$$

$$\therefore \quad \omega_2 = \omega - \frac{3m_b\ell}{m_s L^2}(v_{HIT} + v_b)$$

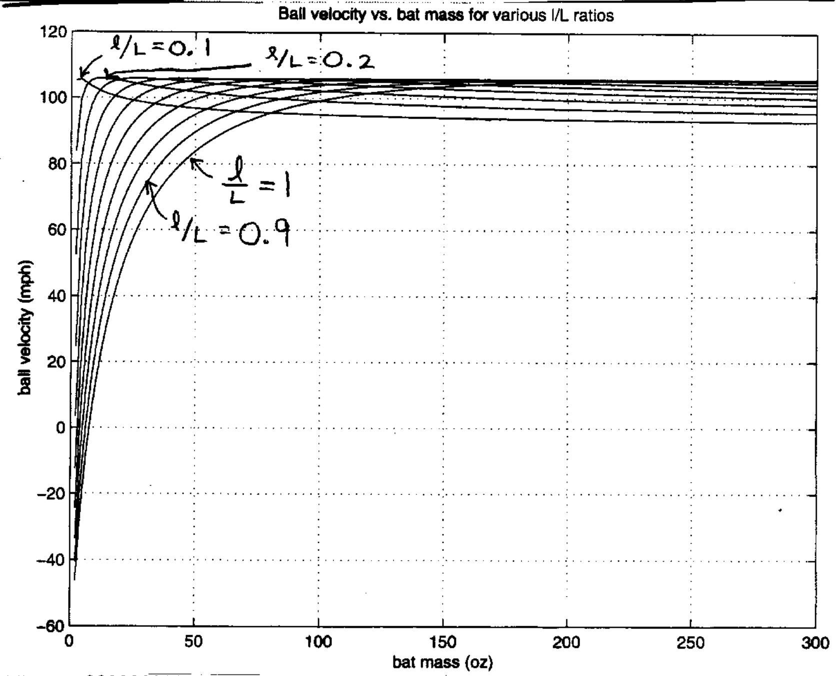
Plugging into (3):

$$\frac{\pi}{2}M_s + \frac{1}{2}m_b v_b^2 = \frac{1}{6}m_s L^2\left[\omega - \frac{3m_b\ell}{m_s L^2}(v_{HIT}+v_b)\right]^2 + \frac{1}{2}m_b v_{HIT}^2$$

Let
$$f(v_{HIT}) = \frac{1}{6}m_s L^2\left[\omega - \frac{3m_b\ell}{m_s L^2}(v_{HIT}+v_b)\right]^2 + \frac{1}{2}m_b v_{HIT}^2 - \left(\frac{\pi}{2}M_s + \frac{1}{2}m_b v_b^2\right)$$

For given values of m_s, ℓ we'd like to find v_{HIT} s.t. $f(v_{HIT}) = 0$. Matlab's 'fzero' function can do this (see code).

The plot on the next page shows v_{HIT} vs. m_s for different ℓ/L ratios. There appear to be several combinations of m_s and ℓ which give a maximum v_{HIT} of about 106 mph. Note that the curves asymptote to 90 mph (the pitch speed) as $m_s \to \infty$. For an infinte bat mass, the bat is basically stationary at impact. So, the ball will rebound w/ the same speed (elastic collision.) (continued)

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% "Solution" to problem 9.61 in HW 12
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

global mb ms Ms L l vb omega

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% All values are converted to SI units for the calculations.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mb=(8/16)*14.5938/32.2;    % bat mass
L=.3048*(32/12);           % length of bat
vb=90*(5280/3600)*0.3048;  % pitch speed
Ms=45;                     % torque on the bat
m=10; n=250;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Determine the values of vhit, omega2, and impulse for
% various values of ms and l.  This is done by calling fzero
% to solve the quadratic equation for vhit.  The ratio of
% the separation to approach speeds are also calculated.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:m
  l=L*(i/m);
  length(i)=l;
  for j=1:n
    ms=(j/4)*mb;
    omega=sqrt(3*pi*Ms/ms)/L;
    mass(i,j)=ms;
    guess=90*(5280/3600)*0.3048;
    vhit(i,j)=fzero('func',guess);
    omega2(i,j)=omega-3*mb*l/(ms*L^2)*(vhit(i,j)+vb);
    impulse(i,j)=mb*(vhit(i,j)+vb);
    app_speed=vb+omega*l;
    sep_speed=-omega2(i,j)*l+vhit(i,j);
    e(i,j)=sep_speed/app_speed;
  end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Convert the mass and velocity to oz and mph.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mass=mass*(32.2*16/14.5938);
vhit=vhit*(3600/5280)/.3048;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot vhit vs. ms for each l/L ratio:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:m
  plot(mass(i,:),vhit(i,:))
  drawnow
  hold on
end

xlabel('bat mass (oz)')
ylabel('ball velocity (mph)')
title('Ball velocity vs. bat mass for various l/L ratios')

   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   % This function takes a value of vhit and finds the value
   % of difference between the final and initial energies.
   % This value is to be made zero for energy conservation.
   %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

   function value=func(vhit)

   global mb ms Ms L l vb omega

   omega2=omega-3*mb*l/(ms*L^2)*(vhit+vb);
   en_initial=pi*Ms/2+mb*vb^2/2;
   en_final=(ms*L^2/6)*omega2^2+(mb/2)*vhit^2;
   value=en_final-en_initial;
```

b) If the bat is too light, it will bounce off the ball and rotate in the opposite direction. Also, the ball may not rebound (in the plot, there are negative ball velocities at small bat masses).

If the bat is too heavy, it would take too much time to swing through 90° compared to the reaction time needed for a 90 mph fastball.

c) Possible aspects of the model which may produce large errors:

① A batter doesn't just rotate his hands to hit a ball. There's also rotation of the hips and arms.

② The collision of the ball with the bat is not perfectly elastic.

③ The hands probably don't impose an impulse during the collision.

d) Possible improvements:

① Put more degrees of freedom into the model for hip and arm movement.

② Allow energy loss by constraining the ratio of separation to approach speeds to be less than 1.

③ Make the applied torque a function of time rather than a constant.

· For those interested in learning more about this problem see

<u>Physics of Baseball</u> by Robert Adair

<u>American Journal of Physics</u> article from 1963 (see Prof. Burns)