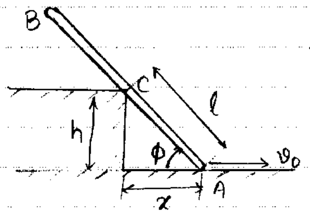
(# 9.5)



v_0 = constant
find $\dot{\phi}, \ddot{\phi}$ in terms of x, h, v_0?
Is $\dot{\phi} > 0$, $\ddot{\phi} > 0$?

The relative velocity equation can be used to solve for almost 2 variables in a 2-D case, so if $\dot{\phi}$ is one of the variables we need another point for which atleast the direction is known which is point C (on the rod)

The velocity is directed along the rod @ point C
$\omega = (-\dot{\phi})\hat{k}$ ($\because$ increasing direction of ϕ is in $-\hat{k}$ direction)

$\vec{v}_C = \vec{v}_A + \vec{\omega} \times \vec{r}_{C/A}$

$v_C(\cos\phi\,\hat{i} - \sin\phi\,\hat{j}) = v_0\hat{i} + -\dot{\phi}\hat{k} \times (-l\cos\phi\,\hat{i} + l\sin\phi\,\hat{j})$

$\Rightarrow v_C(\cos\phi\,\hat{i} - \sin\phi\,\hat{j}) = (v_0 + l\sin\phi\cdot\dot{\phi})\hat{i} + l\dot{\phi}\cos\phi\,\hat{j}$ — ①

① $\cdot\,\hat{i} \Rightarrow v_C\cos\phi = v_0 + l\sin\phi\,\dot{\phi}$ — ②
① $\cdot\,\hat{j} \Rightarrow -v_C\sin\phi = l\dot{\phi}\cos\phi$ — ③

we have $l\cos\phi = x$; $l\sin\phi = h$; $\cot\phi = \frac{x}{h}$

③ $\Rightarrow v_C = \frac{-x\dot{\phi}}{\sin\phi}$

Substituting in ②, $-x\dot{\phi}\cot\phi = v_0 + h\dot{\phi}$
$\Rightarrow x\dot{\phi}\cdot\frac{x}{h} + h\dot{\phi} = -v_0$

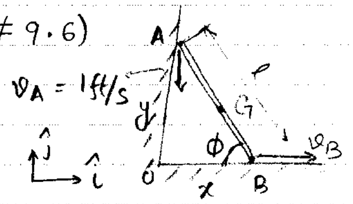
$$\Rightarrow \boxed{\dot{\phi} = \frac{-hv_0}{x^2+h^2}}$$

since $h>0$, $v_0>0$ so $\dot{\phi}<0$ (counterclockwise)

$\ddot{\phi} = \frac{d}{dt}(\dot{\phi}) = \frac{d}{dt}\left(\frac{-hv_0}{x^2+h^2}\right) = +\frac{hv_0\cdot 2x\dot{x}}{(h^2+x^2)^2}$

$\dot{x} = v_0$
$$\Rightarrow \boxed{\ddot{\phi} = \frac{+2xhv_0^2}{(x^2+h^2)^2} > 0}$$

(# 9.6)



find $\vec{v}_G$ when $\phi = 45°$
Given $v_A = -1\hat{j}$

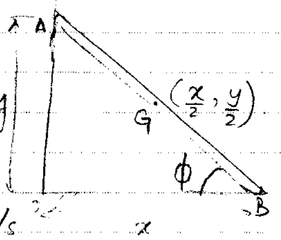
Let us have the coordinate system centered at point O & let x = OB & y = OA

$x^2 + y^2 = l^2$ (a constant)

Differentiating w.r.t time

$2x\dot{x} + 2y\dot{y} = 0$

$\Rightarrow \quad \dot{x} = -\left(\frac{y}{x}\right)\dot{y}$ (ft/s)



Now $\vec{v}_A = \dot{y}\hat{j} = -1\hat{j} \Rightarrow \dot{y} = -1\,ft/s$

when $\phi = 45^\circ$ $\quad y = x$.

So $\dot{x} = -(-1) = 1\,ft/s$

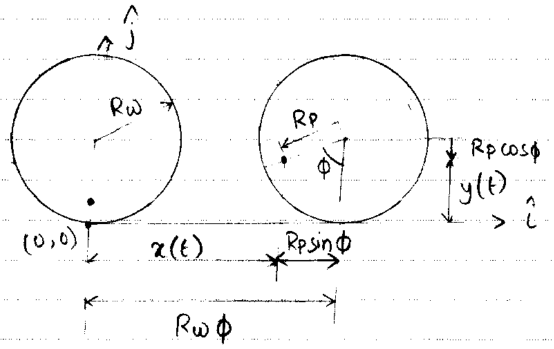
At any point of time $\vec{r}_G = \frac{x}{2}\hat{i} + \frac{y}{2}\hat{j}$

$\vec{v}_G = \frac{d}{dt}(\vec{r}_G) = \frac{\dot{x}}{2}\hat{i} + \frac{\dot{y}}{2}\hat{j}$

$\Rightarrow \boxed{\vec{v}_G = 0.5\,\hat{i} - 0.5\,\hat{j}\ (ft/s)}$

(#9.7)

a)



As can be seen from the figure

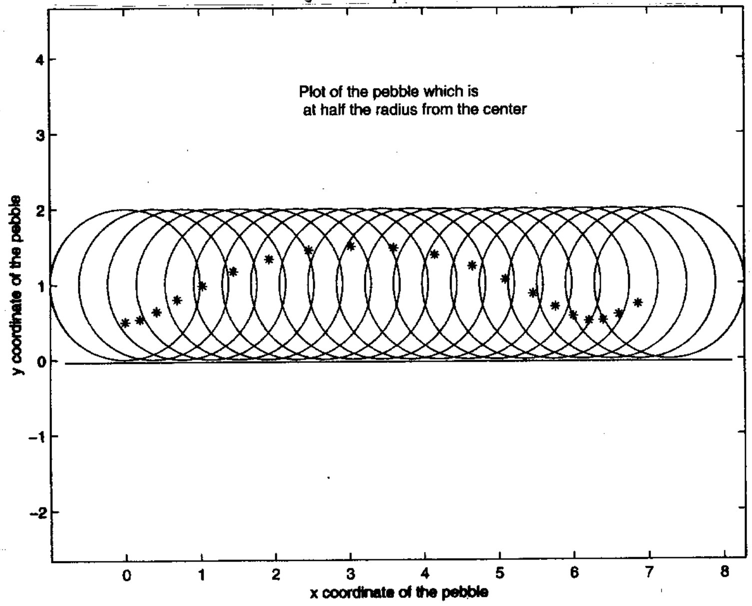
$x(t) = R_w\phi - R_P\sin\phi$

$y(t) = R_w - R_P\cos\phi$

Since $\dot{\phi}$ = constant & $\phi(0) = 0 \quad \therefore \phi = \dot{\phi}t = \omega t$

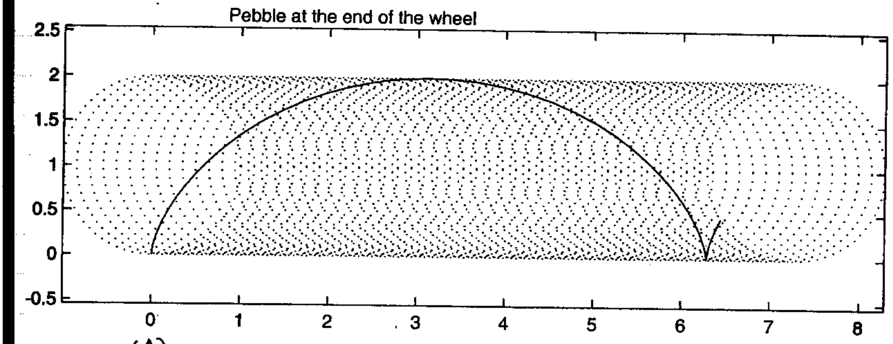
$$\boxed{\begin{aligned} x(t) &= R_w\omega t - R_P\sin(\omega t) \\ y(t) &= R_w - R_P\cos(\omega t) \end{aligned}}$$

b) To plot the path of the pebble with the wheel.



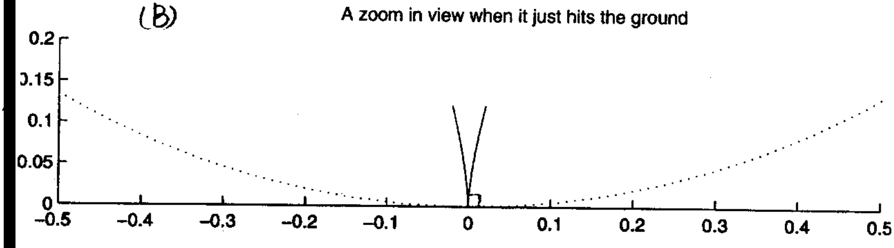
(C) & (d) Now the pebble is assumed to be at the edge of the wheel i.e @ a distance = Rw from the center.

Our claim is that the angle at which the pebble approaches the ground is equal 90°
By making a plot of the path of the pebble near to hitting the ground, we would like to validate our guess.

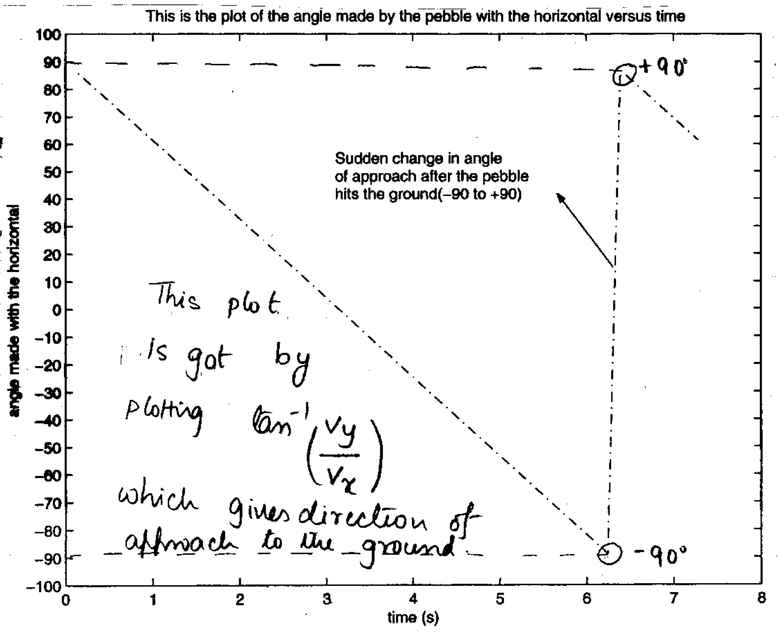


(A)

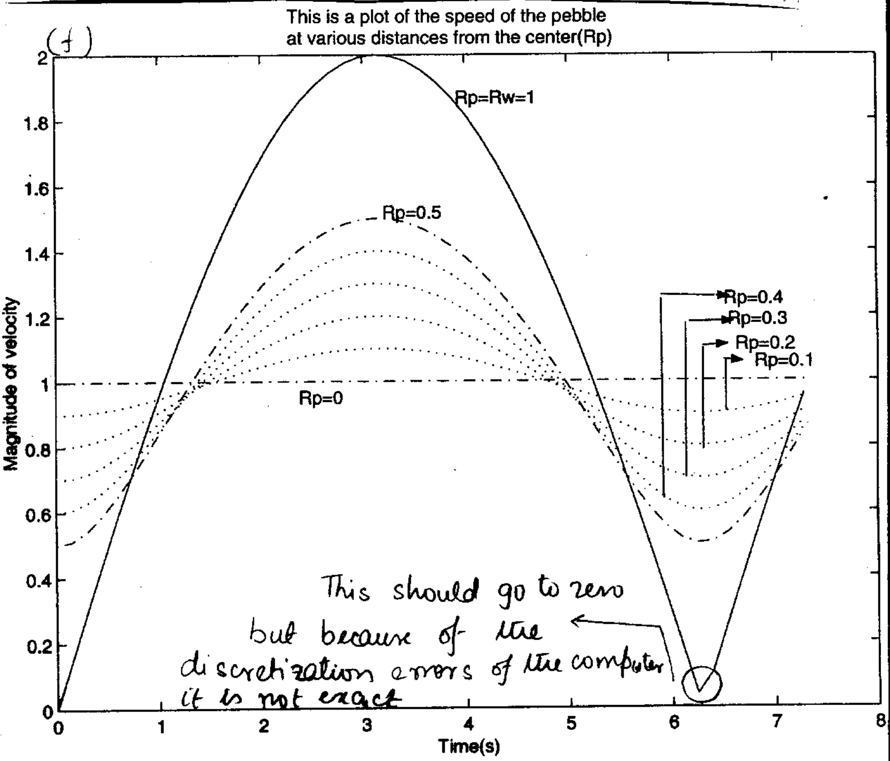
(B)



(e) from plot (B) we can essentially see that the pebble approaches at an angle 90° when it hits the ground.



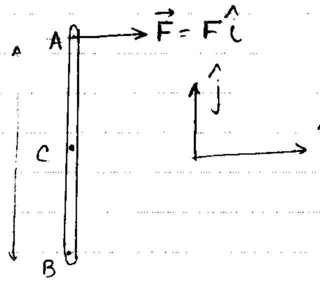
(f) An interesting thing was to plot speed vs time of pebble @ various distances from the centre. Note that the velocity is constant for $R_p = 0$ & when $R_p = R_w$ it is zero →



→ when the pebble hits the ground.

(# 9.22)

FBD:



Stick in Gravity free Space

a) acceleration of C ?

LMB -

$$F\hat{i} = m\vec{a}_{cm} \Rightarrow \boxed{\vec{a}_{cm} = \frac{F}{m}\hat{i}}$$

b) $\dot{\vec{\omega}} = \vec{\alpha} = ?$

AMB about C -

$$\vec{r}_{A/C} \times \vec{F} = \dot{\vec{H}}_C$$

$$\dot{\vec{H}}_C = [I_C]\,\dot{\vec{\omega}} = I^C_{zz}\vec{\alpha} \text{ (in case of 2D motion)}$$

$$= \frac{ml^2}{12}\vec{\alpha}$$

$$\therefore \; -\frac{Fl}{2}\hat{k} = \frac{ml^2}{12}\vec{\alpha}$$

$$\Rightarrow \boxed{\vec{\alpha} = -\frac{6F}{ml}\hat{k}}$$

c) $\vec{a}_A = ?$

Point A & C lie on the same rigid body

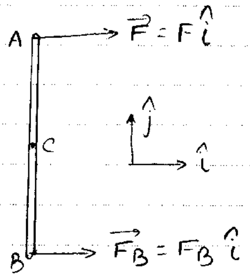
$\therefore \quad \vec{a}_A = \vec{a}_C + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/C}) + \dot{\vec{\omega}} \times \vec{r}_{A/C}$

$\because$ the stick just starts rotation $\vec{\omega} = 0$

$$\vec{a}_A = +\frac{F}{m}\hat{i} + -\frac{6F}{ml}\hat{k} \times \frac{l}{2}\hat{j}$$

$$\boxed{\vec{a}_A = \frac{4F}{m}\hat{i}}$$

(d). What force should be applied @ B so that acceleration of point B = 0



Let F_B be the force

then $\vec{a}_C = \frac{F + F_B}{m}\hat{i}$ (LMB).

AMB about C :-

$$-(F - F_B)\frac{l}{2}\hat{k} = \frac{ml^2}{12}\vec{\alpha}$$

$$\Rightarrow \quad \vec{\alpha} = \frac{(F_B - F).6}{ml}\hat{k}$$

Kinematics relation between B & C

$\vec{a}_B = \vec{a}_C + \vec{\alpha} \times \vec{r}_{B/C}$ ($\vec{\omega} = 0$, just starts from rest)

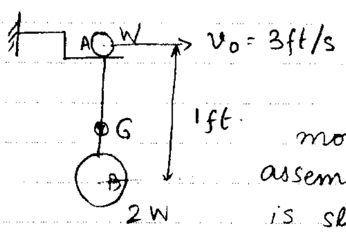
Now $\vec{a}_B = 0$

$$\Rightarrow \quad 0 = \left(\frac{F + F_B}{m}\right)\hat{i} + \frac{(F_B - F)6}{ml}\hat{k} \times -\frac{l}{2}\hat{j}$$

$$0 = \frac{F + F_B + 3F_B - 3F}{m}\hat{i}$$

$$\Rightarrow \quad \boxed{\vec{F}_B = F/2\,\hat{i}}$$

(9.28)

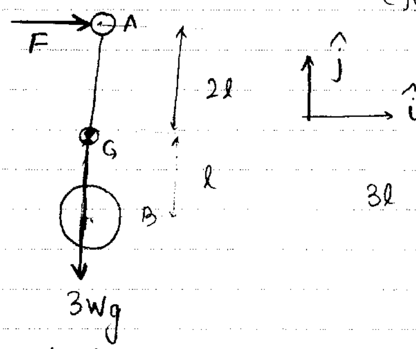


To determine angular momentum of the assembly just after ball A is struck.

Assumption : Since these are "small" spheres, they have negligible moment of inertia about their respective CMs

We can imagine a large force F acting when A is struck (for a very small time)

FBD



$3l = 1ft$

AMB about A:

$$0 = \dot{\vec{H}}_A = \frac{d}{dt}(\vec{H}_A)$$

So $\vec{H}_A$ = constant

$\Rightarrow$ initial Angular Momentum = final Ang. Mom.

initially the system was @ rest

so $\vec{H}_A = 0$

after being struck, let angular velocity of the system be 'ω'

$$\vec{H}_A = \vec{r}_{G/A} \times m\vec{v}_{cm} + I_G \vec{\omega}$$

$$I_G = \frac{W}{g} \cdot 4l^2 + \frac{2W}{g} \cdot l^2 = \frac{6Wl^2}{g}$$

$$\vec{\omega} = \omega_0 \hat{k}, \quad \vec{v}_{cm} = v_{cm}\hat{i}$$

$$\Rightarrow \vec{H}_A = -2l\hat{j} \times \frac{3W}{g} v_{cm}\hat{i} + \frac{6Wl^2}{g}\omega_0 \hat{k} = 0$$

$$\Rightarrow -\cancel{\frac{6Wl}{g}} v_{cm} = \cancel{\frac{6Wl^2}{g}}\omega_0$$

$$\Rightarrow \boxed{v_{cm} = -\omega l} - 1$$

We also know the velocity of A after being hit.

$$\vec{v}_A = v_0\hat{i} = 3\hat{i} \; (ft/s)$$

$$\vec{v}_A = \vec{v}_{cm} + \vec{\omega} \times \vec{r}_{A/CM} = -\omega l\hat{i} + \omega_0\hat{k} \times 2l\hat{j}$$

$$3\hat{i} = -3\omega l\hat{i}$$

$$\Rightarrow \omega = -\frac{3(ft/s)}{1(ft)} = -3\,rad/s$$

Recall ($3l = 1ft$)

$$\vec{\omega} = -3\hat{k}\,(rad/s) \quad \vec{v}_{cm} = 1\hat{i}\,(ft/s)$$

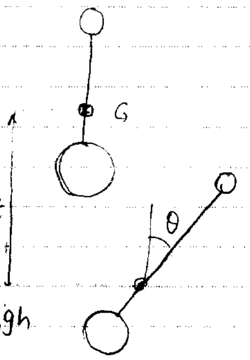
$$\Rightarrow \vec{H}_{CM} = I_G\vec{\omega} = \frac{6Wl^2}{g}\cdot(\omega\hat{k}) = \frac{6\cdot 5}{32}\cdot\frac{1}{9}(-3)\hat{k}\;(lbft s)$$

$$\boxed{\vec{H}_{CM} = -\frac{10}{32}\hat{k}\;(lb\,ft\,s)}$$

a) $\theta = ?$

After the ball A is struck, there are no moments about G

$\Rightarrow \dot{\vec{H}}_G = 0 \Rightarrow \vec{\omega} = \text{constant} = \vec{\omega}_0 = -3\hat{k}\ (\text{rad/s})$



Conserving energy, with initial position of G as 0 datum for P.E

Let final velocity of G be v_G'

$$\tfrac{1}{2}\cancel{I\omega_0^2} + \tfrac{1}{2}m_{tot}v_G^2 = \tfrac{1}{2}\cancel{I\omega_0^2} + \tfrac{1}{2}m_{tot}(v_G')^2 - m_{tot}gh$$

$$\Rightarrow v_G^2 + 2gh = v_G'^2 \qquad (v_G = 1\,ft/s)$$

$$\Rightarrow v_G'^2 = 1(ft/s)^2 + 2\times 32(ft/s^2)\times 2(ft)$$

$$\Rightarrow v_G' = \sqrt{129}\ ft/s$$

Since there is uniform downward acceleration the time taken to drop by 2ft

$\frac{1}{2}gt^2 = h$ ($\because$ initially there is no vertical velocity)

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{4\,ft}{32\,ft/s^2}} = \frac{1}{2\sqrt{2}}\ sec = 0.354\,sec$$

$$\therefore \boxed{\theta = \omega t = 3(rad/s)\times\frac{1}{\sqrt{2}}(s) = 1.061\,rad}$$

b) $\vec{v}_A = ?$

$$\vec{v}_A = \vec{v}_G' + \vec{\omega}\times\vec{r}_{A/G}$$

$$\vec{r}_{A/G} = \tfrac{2}{3}(\cos\theta\,\hat{j} + \sin\theta\,\hat{i})$$

$$\vec{v}_G' = -gt\,\hat{j} + 1(ft/s)\,\hat{i} = -11.31\,\hat{j} + 1\,\hat{i}\ (ft/s)$$

$$\vec{v}_A = -11.31\,\hat{j} + \hat{i} + 0.976\,\hat{i} - 1.746\,\hat{j}\ (ft/s)$$

$$\Rightarrow \boxed{\vec{v}_A = -13.056\,\hat{j} + 1.976\,\hat{i}\ (ft/s)}$$

(c) Total KE of the assembly

$$E_K = \tfrac{1}{2}mv_G'^2 + \tfrac{1}{2}I\omega_0^2$$

$$= \tfrac{1}{2}\cdot\frac{3W}{g}v_G'^2 + \tfrac{1}{2}\frac{6Wl^2}{g}\omega_0^2$$

$$= \tfrac{1}{2}\cdot\frac{15}{32}\cdot 129 + \tfrac{1}{2}\cdot\frac{30}{32}\cdot\tfrac{1}{9}\cdot 9\ (lbft)$$

$$\boxed{E_K = 30.70\ lbft}$$

(d) $\vec{a}_A = ?$

$$\vec{a}_A = \vec{a}_G + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/G}) + \underbrace{\cancel{\dot{\vec{\omega}}}}_{=0}\times\vec{r}_{A/G}$$

$$= -32\,\hat{j} + (-2.928)\,\hat{j} - 5.238\,\hat{i}\ (ft/s^2)$$

$$\boxed{\vec{a}_A = -5.238\,\hat{i} - 34.928\,\hat{j}\ (ft/s^2)}$$