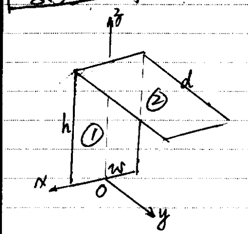
HW #10.

#8.53



$w = 200mm,\ h = 400mm.$
$d = 300mm.$

two parts welded together called ① & ②

$[I^0] = [I^0]_1 + [I^0]_2$

From parallel-axis theorem,

we get $[I^0]_1 = [I^{CM}]_1 + m_1 \begin{bmatrix} y_G^2+z_G^2 & -x_G y_G & -x_G z_G \\ -x_G y_G & x_G^2+z_G^2 & -y_G z_G \\ -x_G z_G & -y_G z_G & x_G^2+y_G^2 \end{bmatrix}$

since it's a uniform plate.

$\therefore m_1 = \frac{m}{h+d} h.$

$$\therefore [I^0]_1 = \frac{mh}{12(h+d)} \begin{bmatrix} h^2 & 0 & 0 \\ 0 & w^2+h^2 & 0 \\ 0 & 0 & w^2 \end{bmatrix} + \frac{mh}{h+d} \begin{bmatrix} \frac{h^2}{4} & 0 & 0 \\ 0 & \frac{h^2}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

use the table at book back

$$= \frac{mh}{h+d} \begin{bmatrix} \frac{1}{3}h^2 & 0 & 0 \\ 0 & \frac{h^2}{3}+\frac{w^2}{12} & 0 \\ 0 & 0 & \frac{w^2}{12} \end{bmatrix}$$

Similarly, $[I^0]_2 = [I^{CM}]_2 + m_2 \begin{bmatrix} y_G^2+z_G^2 & -x_G y_G & -x_G z_G \\ -x_G y_G & x_G^2+z_G^2 & -y_G z_G \\ -x_G z_G & -y_G z_G & x_G^2+y_G^2 \end{bmatrix}$

$$= \frac{md}{12(h+d)} \begin{bmatrix} d^2 & 0 & 0 \\ 0 & w^2 & 0 \\ 0 & 0 & w^2+d^2 \end{bmatrix} + \frac{md}{h+d} \begin{bmatrix} \frac{d^2}{4}+h^2 & 0 & \\ 0 & h^2 & \\ 0 & -\frac{dh}{2} & \end{bmatrix}$$

$$= \frac{md}{h+d} \begin{bmatrix} \frac{1}{3}d^2+h^2 & 0 & 0 \\ 0 & \frac{1}{12}w^2+h^2 & -\frac{1}{2}dh \\ 0 & -\frac{1}{2}dh & \frac{1}{3}d^2+\frac{w^2}{12} \end{bmatrix}$$

$$\therefore [I^0] = \frac{m}{h+d} \begin{bmatrix} \frac{1}{3}h^3+\frac{1}{3}d^3+h^2d & 0 & 0 \\ 0 & \frac{hw^2}{12}+\frac{h^3}{3}+\frac{w^2d}{12}+dh^2 & -\frac{1}{2}d^2h \\ 0 & -\frac{d^2}{2}h & \frac{1}{12}w^2h+\frac{1}{12}w^2d+\frac{d^3}{3} \end{bmatrix}$$

8.55

Find angle between $\vec{\omega} = 3\ rad/s\ \hat{k}$ & $\vec{H} = [I^G]\vec{\omega}$

$[I^G] = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 40 \end{bmatrix} kg\cdot m^2$

$\therefore \vec{H} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 40 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 120 \end{bmatrix} kg\cdot m^2/s\ \hat{k}$

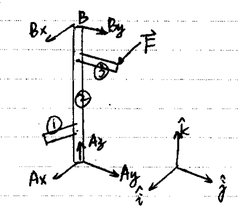
$\therefore \vec{H}$ is paralell to $\vec{\omega}$

ie, angle is 0.

8.63

neglect gravity.

FBD



$\Sigma \vec{M_A} = \vec{\omega} \times \vec{H_A} + [I_A]\cdot\dot{\vec{\omega}}$

At the beginning $\vec{\omega} = 0$. $\therefore \Sigma\vec{M_A} = [I_A]\cdot\dot{\vec{\omega}}$

$[I_A^{①}] = \begin{bmatrix} \frac{1}{16}ml^2 & 0 & -\frac{1}{8}mld \\ 0 & \frac{ml^2}{16}+\frac{md^2}{3} & 0 \\ -\frac{1}{8}mld & 0 & \frac{1}{3}md^2 \end{bmatrix}$

$[I_A^{②}] = \begin{bmatrix} \frac{1}{3}ML^2 & 0 & 0 \\ 0 & \frac{1}{3}ML^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$[I_A^{③}] = \begin{bmatrix} \frac{9}{16}ml^2+\frac{1}{3}md^2 & 0 & 0 \\ 0 & \frac{9}{16}ml^2 & -\frac{3}{8}mld \\ 0 & -\frac{3}{8}mld & \frac{1}{3}md^2 \end{bmatrix}$

$\therefore [I_A] = \begin{bmatrix} \frac{5}{8}ml^2+\frac{2}{3}md^2+\frac{1}{3}ML^2 & 0 & -\frac{1}{8}mld \\ 0 & \frac{5}{8}ml^2+\frac{1}{3}md^2+\frac{1}{3}ML^2 & -\frac{3}{8}mld \\ -\frac{1}{8}mld & -\frac{3}{8}mld & \frac{2}{3}md^2 \end{bmatrix}$

$\therefore [I_A]\cdot\dot{\vec{\omega}} = [I_A]\cdot\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} = -\frac{1}{8}mld\alpha\hat{i} - \frac{3}{8}mld\alpha\hat{j} + \frac{2}{3}md^2\hat{k}$

$\Sigma\vec{M_A} = \vec{r}\times\vec{F} + \vec{AB}\times\vec{B_x} + \vec{AB}\times\vec{B_y}$

$= (d\hat{j} + \frac{3}{4}l\hat{k})\times(-F\hat{i}+G\hat{k}) + l\hat{k}\times B_x\hat{i} + l\hat{k}\times B_y\hat{j}$

$= Fd\hat{k} - \frac{3}{4}Fl\hat{j} + Gd\hat{i} + B_x l\hat{j} - B_y l\hat{i}$

$\therefore \{\ \}\cdot\hat{k}: \Rightarrow \frac{2}{3}md\alpha = Fd$ $\quad \therefore \boxed{\alpha = \frac{3F}{2md}}$

8.63 cont'd.

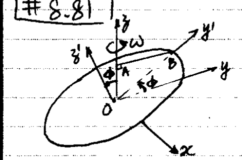
And $\{\ \}\cdot\hat{j}: \Rightarrow B_x l - \frac{3}{4}Fl = -\frac{3}{8}mld\alpha = -\frac{9}{16}Fl$

$\therefore \boxed{B_x = \frac{3}{16}F}$

$\{\ \}\cdot\hat{i} \Rightarrow Gd - B_y l = -\frac{1}{8}mld\alpha = -\frac{3}{16}Fl$

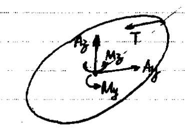
$\therefore \boxed{B_y = \frac{d}{l}G + \frac{3}{16}F}$

#8.81



massless wire perpendicula to the shaft.

FBD:



Consider the x'y'z' coordinate system

$\hat{i} = \hat{i}',\quad \hat{j}' = \cos\phi\hat{j} + \sin\phi\hat{k},\quad \hat{k}' = -\sin\phi\hat{j} + \cos\phi\hat{k}$

And $[I_O]_{x'y'z'} = \begin{bmatrix} \frac{1}{4}mR^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{bmatrix}$

since $\vec{\omega}$ is a constant,

$\therefore \dot{\vec{H_o}} = \vec{\omega}\times\vec{H_o}$, $\vec{H_o} = [I_o]_{x'y'z'}\cdot\vec{\omega}_{x'y'z'}$

$\vec{\omega}_{x'y'z'} = \omega\hat{k} = \omega\sin\phi\hat{j}' + \omega\cos\phi\hat{k}'$

$\therefore \vec{H_o} = \begin{bmatrix} \frac{1}{4}mR^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{bmatrix}\begin{bmatrix} 0 \\ \omega\sin\phi \\ \omega\cos\phi \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{4}mR^2\omega\sin\phi \\ \frac{1}{2}mR^2\omega\cos\phi \end{bmatrix}$

$\therefore \dot{\vec{H_o}} = (\omega\sin\phi\hat{j}' + \omega\cos\phi\hat{k}')\times(\frac{1}{4}mR^2\omega\sin\phi\hat{j}' + \frac{1}{2}mR^2\omega\cos\phi\hat{k}')$

$= \frac{1}{2}mR^2\omega^2\sin\phi\cos\phi\hat{i}' - \frac{1}{4}mR^2\omega^2\sin\phi\cos\phi\hat{i}'$

$= \frac{1}{4}mR^2\omega^2\sin\phi\cos\phi\hat{i}'$

$= \frac{1}{4}mR^2\omega^2\sin\phi\cos\phi\hat{i}$

And, $\Sigma\vec{M_o} = \vec{r}\times\vec{T} + M_y\hat{j} + M_z\hat{k}$

$= R\hat{j}'\times(-T\cos\phi\hat{j}' + T\sin\phi\hat{k}') + M_y\hat{j} + M_z\hat{k}$

$= TR\sin\phi\,\hat{i}' + M_y\hat{j} + M_z\hat{k}$

$= TR\sin\phi\,\hat{i} + M_y\hat{j} + M_z\hat{k}$

8.81 Cont'd

$\{\ \}\cdot\hat{i}:\quad TR\sin\phi = \frac{1}{4}mR^2\omega^2\sin\phi\cos\phi$

$\therefore \boxed{T=\frac{1}{4}mR\omega^2\cos\phi}$

$\boxed{\#8.92}$

$\vec{\omega}$ is a constant. $\therefore \dot{\vec{H}}_0 = \vec{\omega}\times\vec{H}_0$

$\therefore$ When $\dot{\vec{H}}=0$, $\vec{H}_0=[I^0]\vec{\omega}$ must be parallel to the angular velocity $\vec{\omega}$. i.e. $\vec{\omega}$ is the eigenvector of $[I^0]$. Also the center of mass must lie on the axis of rotation.

a) Yes.

$\vec{\omega}$ is parallel to $\vec{H}_0$ $\therefore \dot{\vec{H}}_0=0$ $[I^0]=\frac{m}{12}\begin{bmatrix} a^2+b^2 & 0 & \\ 0 & a^2 & \\ 0 & 0 & \end{bmatrix}$

b) yes.

$\dot{\vec{H}}_0=0$. $(\vec{\omega}\parallel\vec{H}_0)$

c). No.

mass center isn't on the rotation axis. $\dot{\vec{H}}_0\neq0$

d). yes.

$[I^0]=\begin{bmatrix} xx & xx & 0 \\ xx & xx & 0 \\ 0 & 0 & xx \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}0\\0\\\omega\end{bmatrix}$ $\therefore \vec{H}_0=\begin{bmatrix}0\\0\\xx\end{bmatrix}\parallel$

e). yes.

$[I^0]=\begin{bmatrix} xx & 0 & 0 \\ 0 & xx & 0 \\ 0 & 0 & xx \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}0\\0\\\omega\end{bmatrix}$ $\therefore \vec{H}_0=\begin{bmatrix}0\\0\\xx\end{bmatrix}\parallel\vec{\omega}$

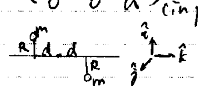
f). yes.

$[I^0]=\frac{2}{5}mR^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}0\\0\\\omega\end{bmatrix}$ $\vec{H}_0\parallel\vec{\omega}$

g) yes

$[I^0]=\frac{1}{6}m\begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}$ $\vec{H}_0=\frac{ma^2}{6}\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}\parallel\vec{\omega}$ (in principal axis coordinates)

h) No.



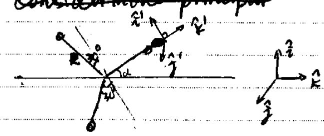
$[I^0]=\begin{bmatrix} 2md^2 & \cancel{2mdR}\ 0 & 2mdR \\ 0 & 2md^2+2mR^2 & 0 \\ 2mdR & 0 & 2mR^2 \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}0\\0\\\omega\end{bmatrix}$

$\vec{H}_0=2m\omega\begin{bmatrix}dR\\0\\R^2\end{bmatrix}\nparallel\vec{\omega}$

#8.92 Cont'd.

(i). Yes.

~~Consider the principal~~



in $x'y'z'$ coordinate system.

$[I^0]=\begin{bmatrix} \frac{3}{2}mR^2 & 0 & 0 \\ 0 & 3mR^2 & 0 \\ 0 & 0 & \frac{3}{2}mR^2 \end{bmatrix}$ $\vec{\omega}=\begin{bmatrix}-\omega\sin\alpha\\0\\\omega\cos\alpha\end{bmatrix}$

$\vec{H}_0=\frac{3}{2}mR^2\omega\begin{bmatrix}-\sin\alpha\\0\\\cos\alpha\end{bmatrix}\parallel\vec{\omega}$

(j). yes.

Since rotation axis is perpendicular to the two eigenvectors of $[I^0]$, it must parallel to the third eigenvector of $[I^0]$. which means $\vec{H}_0\parallel\vec{\omega}$ $\therefore \vec{\omega}\times\vec{H}=0$. $\dot{\vec{H}}=0$