Your Name: ______________________

Your TA: ______________________

T&AM 203 Prelim 1

Tuesday February 29, 2000 7:30 — 9:00$^+$ PM

Draft February 27, 2000

3 problems, 100 points, and 90$^+$ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. Two pages of formulas, from the front of the text, and a blank page for tentative scrap work are provided at the back. Ask for extra scrap paper if you need it.

b) Full credit if

- →free body diagrams← are drawn whenever linear or angular momentum balance is used;
- correct vector notation is used, when appropriate;

↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
± all signs and directions are well defined with sketches and/or words;

→| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);

□ your answers are $\boxed{\text{boxed}}$ in; and

≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot\theta_7$ `= 18`" instead of, say, "`theta7dot = 18`".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

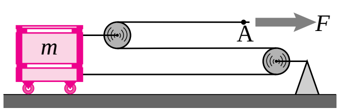
Problem	1:	/30
Problem	2:	/35
Problem	3:	/35
TOTAL:		**/100**

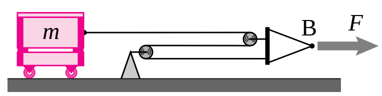
1) (30 pts) The problems below are independent. MATLAB commands are not allowed except in part (c).

1a) (10 pts) A mass m is connected to a spring k and released from rest with the spring stretched a distance d from its static equilibrium position. It then oscillates back and forth repeatedly crossing the equilibrium. How much time passes from release until the mass moves through the equilibrium position for the second time? (Answer in terms of some or all of m, k, and d.) [Neglect gravity and friction.]

Comparison

1b) (10 pts) Assuming equal masses and equal forces in the two cases, what is the ratio of the acceleration of point A to that of point B? [assume massless ideal pulleys etc]





1c) (10 pts) In three-dimensional space with no gravity a particle with $m = 3\,\text{kg}$ at A is pulled by three strings which pass through points B, C, and D respectively. The acceleration is known to be $\underline{\mathbf{a}} = (1\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}})\,\text{m/s}^2$. The position vectors of B, C, and D relative to A are given in the first few lines of code below. Complete the code to find the three tensions. The last line should read `T = ...` with `T` being assigned to be a 3-element column vector with the three tensions in Newtons. [Hint: If `x`, `y`, and `z` are three column vectors then `A=[x y z]` is a matrix with `x`, `y`, and `z` as columns.]

```
% a MATLAB script file to find 3 tensions
m   = 3;
a   = [ 1 2 3]';

rAB = [ 2 3 5]';
rAC = [-3 4 2]';
rAD = [ 1 1 1]';

uAB = rAB/norm(rAB); % norm gives vector magnitude
% You write the code below (4 to 5 lines).
% Don't copy any of the numbers above.
% Don't do any arithmetic on the side.
```

2)(35 pts) A particle of mass m moves in a viscous fluid which resists motion with a force of magnitude $F = c|\underline{\mathbf{v}}|$, where $\underline{\mathbf{v}}$ is the velocity. Do not neglect gravity.

a) (10 ptss) In terms of some or all of g, m, and c, what is the particle's terminal (steady-state) falling speed?

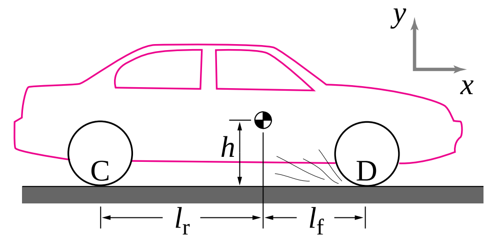
b) (15 pts) Starting with a free body diagram and linear momentum balance, find two second order scalar differential equations that describe the two-dimensional motion of the particle.

c) (10 pts) (challenge, do last, long calculation) Assume the particle is thrown from $\underline{\mathbf{r}} = \underline{\mathbf{0}}$ with $\underline{\mathbf{v}} = v_{x0}\,\hat{\mathbf{i}} + v_{y0}\,\hat{\mathbf{j}}$ at a vertical wall a distance d away. Find the height h along the wall where the particle hits. (Answer in terms of some or all of v_{x0}, v_{y0}, m, g, c, and d.)

[Hint: i) find $x(t)$ and $y(t)$ like in the homework, ii) eliminate t, iii) substitute $x = d$. The answer is not tidy. In the limit $d \to 0$ the answer reduces to a sensible dependence on d (The limit $c \to 0$ is also sensible.). If you use Matlab, start your code by assigning any non-trivial values to all constants.]

Problem 2 continued.

3) 35 pts) Car accelerating. A car (mass $=m$) with a big motor, front-wheel drive, and a stiff suspension accelerates to the right with the front wheels over-powered and skidding (friction coefficient $=\mu$) and back wheels turning freely.



a) (5 pts) Assuming the car starts from rest and has constant acceleration a, how far has it travelled in time t? (Answer in terms of a and t.) [Not a trick, just easy.].

b) (30 pts) Find a in terms of any or all of ℓ_r, ℓ_f, h, m, g and μ. [Hint: all the directions on the cover page apply. Your answer should reduce to $a = \ell_r g/h$ in the limit $\mu \to \infty$.]

Problem 3 continued.

Summary of Mechanics

0) **The laws of mechanics apply to any collection of material or 'body.'** This body could be the overall system of study or any part of it. In the equations below, the forces and moments are those that show on a free body diagram. Interacting bodies cause equal and opposite forces and moments on each other.

I) Linear Momentum Balance (LMB)/Force Balance

Equation of Motion	$\boxed{\sum \vec{F}_i = \dot{\vec{L}}}$	The total force on a body is equal to its rate of change of linear momentum.	(I)
Impulse-momentum (integrating in time)	$\int_{t_1}^{t_2} \sum \vec{F}_i \cdot dt = \Delta \vec{L}$	Net impulse is equal to the change in momentum.	(Ia)
Conservation of momentum (if $\sum \vec{F}_i = \vec{0}$)	$\dot{\vec{L}} = \vec{0} \quad \Rightarrow$ $\Delta \vec{L} = \vec{L}_2 - \vec{L}_1 = \vec{0}$	When there is no net force the linear momentum does not change.	(Ib)
Statics (if $\dot{\vec{L}}$ is negligible)	$\sum \vec{F}_i = \vec{0}$	If the inertial terms are zero the net force on system is zero.	(Ic)

II) Angular Momentum Balance (AMB)/Moment Balance

Equation of motion	$\boxed{\sum \vec{M}_{\mathrm{C}} = \dot{\vec{H}}_{\mathrm{C}}}$	The sum of moments is equal to the rate of change of angular momentum.	(II)
Impulse-momentum (angular) (integrating in time)	$\int_{t_1}^{t_2} \sum \vec{M}_{\mathrm{C}} dt = \Delta \vec{H}_{\mathrm{C}}$	The net angular impulse is equal to the change in angular momentum.	(IIa)
Conservation of angular momentum (if $\sum \vec{M}_{\mathrm{C}} = \vec{0}$)	$\dot{\vec{H}}_{\mathrm{C}} = \vec{0} \quad \Rightarrow$ $\Delta \vec{H}_{\mathrm{C}} = \vec{H}_{\mathrm{C2}} - \vec{H}_{\mathrm{C1}} = \vec{0}$	If there is no net moment about point C then the angular momentum about point C does not change.	(IIb)
Statics (if $\dot{\vec{H}}_{\mathrm{C}}$ is negligible)	$\sum \vec{M}_{\mathrm{C}} = \vec{0}$	If the inertial terms are zero then the total moment on the system is zero.	(IIc)

III) Power Balance (1st law of thermodynamics)

Equation of motion	$\dot{Q} + P = \underbrace{\dot{E}_{\mathrm{K}} + \dot{E}_{\mathrm{P}} + \dot{E}_{\mathrm{int}}}_{\dot{E}}$	Heat flow plus mechanical power into a system is equal to its change in energy (kinetic + potential + internal).	(III)
for finite time	$\int_{t_1}^{t_2} \dot{Q} dt + \int_{t_1}^{t_2} P dt = \Delta E$	The net energy flow going in is equal to the net change in energy.	(IIIa)
Conservation of Energy (if $\dot{Q} = P = 0$)	$\dot{E} = 0 \quad \Rightarrow$ $\Delta E = E_2 - E_1 = 0$	If no energy flows into a system, then its energy does not change.	(IIIb)
Statics (if $\dot{E}_{\mathrm{K}}$ is negligible)	$\dot{Q} + P = \dot{E}_{\mathrm{P}} + \dot{E}_{\mathrm{int}}$	If there is no change of kinetic energy then the change of potential and internal energy is due to mechanical work and heat flow.	(IIIc)
Pure Mechanics (if heat flow and dissipation are negligible)	$\boxed{P = \dot{E}_{\mathrm{K}} + \dot{E}_{\mathrm{P}}}$	In a system well modeled as purely mechanical the change of kinetic and potential energy is due to mechanical work.	(IIId)

Some Definitions

(Please also look at the tables inside the back cover.)

$\vec{\boldsymbol{r}}$ or $\vec{\boldsymbol{x}}$	Position	(*e.g.*, $\vec{\boldsymbol{r}}_i \equiv \vec{\boldsymbol{r}}_{i/\mathrm{O}}$ is the position of a point i relative to the origin, O)
$\vec{\boldsymbol{v}} \equiv \frac{d\vec{\boldsymbol{r}}}{dt}$	Velocity	(*e.g.*, $\vec{\boldsymbol{v}}_i \equiv \vec{\boldsymbol{v}}_{i/\mathrm{O}}$ is the velocity of a point i relative to O, measured in a non-rotating reference frame)
$\vec{\boldsymbol{a}} \equiv \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{d^2\vec{\boldsymbol{r}}}{dt^2}$	Acceleration	(*e.g.*, $\vec{\boldsymbol{a}}_i \equiv \vec{\boldsymbol{a}}_{i/\mathrm{O}}$ is the acceleration of a point i relative to O, measured in a Newtonian frame)
$\vec{\boldsymbol{\omega}}$	Angular velocity	A measure of rotational velocity of a rigid body.
$\vec{\boldsymbol{\alpha}} \equiv \dot{\vec{\boldsymbol{\omega}}}$	Angular acceleration	A measure of rotational acceleration of a rigid body.
$\vec{\boldsymbol{L}} \equiv \begin{cases} \sum \mathrm{m}_i \vec{\boldsymbol{v}}_i & \text{discrete} \\ \int \vec{\boldsymbol{v}}\, dm & \text{continuous} \end{cases}$ $= m_{\mathrm{tot}} \vec{\boldsymbol{v}}_{\mathrm{cm}}$	Linear momentum	A measure of a system's net translational rate (weighted by mass).
$\dot{\vec{\boldsymbol{L}}} \equiv \begin{cases} \sum \mathrm{m}_i \vec{\boldsymbol{a}}_i & \text{discrete} \\ \int \vec{\boldsymbol{a}}\, dm & \text{continuous} \end{cases}$ $= m_{\mathrm{tot}} \vec{\boldsymbol{a}}_{\mathrm{cm}}$	Rate of change of linear momentum	The aspect of motion that balances the net force on a system.
$\vec{\boldsymbol{H}}_{\mathrm{C}} \equiv \begin{cases} \sum \vec{\boldsymbol{r}}_{i/\mathrm{C}} \times \mathrm{m}_i \vec{\boldsymbol{v}}_i & \text{discrete} \\ \int \vec{\boldsymbol{r}}_{/C} \times \vec{\boldsymbol{v}}\, dm & \text{continuous} \end{cases}$	Angular momentum about point C	A measure of the rotational rate of a system about a point C (weighted by mass and distance from C).
$\dot{\vec{\boldsymbol{H}}}_{\mathrm{C}} \equiv \begin{cases} \sum \vec{\boldsymbol{r}}_{i/\mathrm{C}} \times \mathrm{m}_i \vec{\boldsymbol{a}}_i & \text{discrete} \\ \int \vec{\boldsymbol{r}}_{/C} \times \vec{\boldsymbol{a}}\, dm & \text{continuous} \end{cases}$	Rate of change of angular momentum about point C	The aspect of motion that balances the net torque on a system about a point C.
$E_{\mathrm{K}} \equiv \begin{cases} \frac{1}{2} \sum \mathrm{m}_i v_i^2 & \text{discrete} \\ \frac{1}{2} \int v^2\, dm & \text{continuous} \end{cases}$	Kinetic energy	A *scalar* measure of net system motion.
$E_{int} =$ (heat-like terms)	Internal energy	The non-kinetic non-potential part of a system's total energy.
$P \equiv \sum \vec{\boldsymbol{F}}_i \cdot \vec{\boldsymbol{v}}_i + \sum \vec{\boldsymbol{M}}_i \cdot \vec{\boldsymbol{\omega}}_i$	Power of forces and torques	The mechanical energy flow into a system. Also, $P \equiv \dot{W}$, rate of work.
$[\boldsymbol{I}^{\mathrm{cm}}] \equiv \begin{bmatrix} I_{xx}^{cm} & I_{xy}^{cm} & I_{xz}^{cm} \\ I_{xy}^{cm} & I_{yy}^{cm} & I_{yz}^{cm} \\ I_{xz}^{cm} & I_{yz}^{cm} & I_{zz}^{cm} \end{bmatrix}$	Moment of inertia matrix about cm	A measure of how mass is distributed in a rigid body.