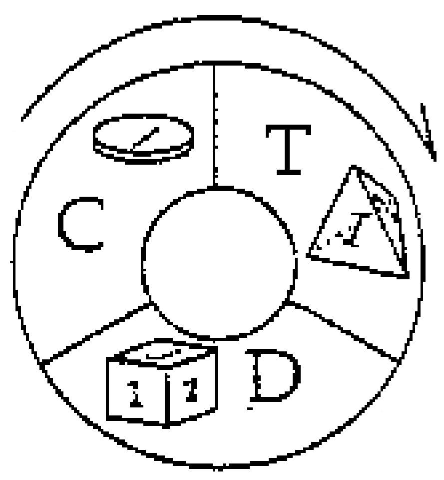
.Finite Diffence Equations Section 2.6

M294 P III #4

(25 pt) The three "spaces" on the simple board game shown are labeled "C", "T", and "D" for coin, tetrahedron, and dice. On one turn a player advances clockwise a random number of spaces as determined by shaking and dropping the object on their present space (From the C position a player moves 1 or 2 spaces with equal probabilities, from the T space a player moves 1-4 spaces with equal probabilities, and from the D space a player moves 1-6 spaces with equal probabilities.).

In very long games what fraction of the moves end up on the D space on average? [Hint: Use exact arithmetic rather than truncated decimal representations. The needed calculations easily fit in the space alotted.]

Gameboard



Hint:
(If you use this table, briefly define the entries.)

$$\begin{bmatrix}\text{Probability of} \\ \text{switch from} \\ j \text{ to } i \text{ if} \\ \text{at } j.\end{bmatrix} = A_{ij}, \quad A = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \\ C & T & D \end{array} \\ \begin{bmatrix} 0 & 1/4 & 1/3 \\ 1/2 & 1/4 & 1/3 \\ 1/2 & 1/2 & 1/3 \end{bmatrix} \end{array} \begin{array}{cc} C & 1 \\ T & 2 \\ D & 3 \end{array}$$

e.g. probability of getting to D given a start at T is $1/4 + 1/4 = 1/2$

⇐ *Please put scrap work for problem 4 on the page to the left* ⇐.
⇓ *Put neat work to be graded for problem 4 below.* ⇓
(*If you need the space, clearly mark work to be graded on the scrap page.*)

Let $\underline{v}^n = \begin{bmatrix} v_C^n \\ v_T^n \\ v_D^n \end{bmatrix}$ = probabilities of being on squares C, T & D at move n.

Basic Markov eq.

$\boxed{\underline{v}^{n+1} = A\underline{v}^n}$ ⇒ long term steady state is $A\underline{v} = \underline{v}$

⇒ $(A - I)\underline{v} = \underline{0}$. Solve for $\underline{v}$ by row operations...

$$\begin{bmatrix} -1 & 1/4 & 1/3 \\ 1/2 & -3/4 & 1/3 \\ 1/2 & 1/2 & -2/3 \end{bmatrix} \sim \begin{bmatrix} -1 & 1/4 & 1/3 \\ 0 & -5/8 & 1/2 \\ 0 & 5/8 & -1/2 \end{bmatrix} \sim \begin{bmatrix} -1 & 1/4 & 1/3 \\ 0 & -5/8 & 1/2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \boxed{V_D = S}$$

free

$$-V_C + \tfrac{1}{4}V_T + \tfrac{1}{3}S = 0 \qquad \Rightarrow -V_C + \tfrac{1}{5}S + (\tfrac{1}{3})S = 0 \Rightarrow \boxed{V_C = \tfrac{8}{15}S}$$

$$-\tfrac{5}{8}V_T + \tfrac{1}{2}S = 0 \Rightarrow \boxed{V_T = \tfrac{4}{5}S}$$

$$\Rightarrow \underline{V} = S\begin{bmatrix} 8/15 \\ 4/5 \\ 1 \end{bmatrix} = t\begin{bmatrix} 8 \\ 12 \\ 15 \end{bmatrix}$$

but for sensible probability need $\boxed{V_C + V_T + V_D = 1}$

$$\Rightarrow t = \frac{1}{8+12+15} = \frac{1}{35}$$

$$\Rightarrow \underline{V} = \begin{bmatrix} V_C \\ V_T \\ V_D \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 8 \\ 12 \\ 15 \end{bmatrix} \Rightarrow$$

prob. of being on D in the long run

$$= V_D = \frac{15}{35} = \frac{3}{7}$$