Coordinates

M294 P II FA98 #3

0) (25 pt) Besides the standard basis $\mathcal{E}$ here are two bases for $\mathcal{R}^2$:

$$\mathcal{B} = \left\{ \underbrace{\begin{bmatrix}1\\1\end{bmatrix}}_{b_1}, \underbrace{\begin{bmatrix}-1\\1\end{bmatrix}}_{b_2} \right\} \text{ and } \mathcal{C} = \left\{ \underbrace{\begin{bmatrix}2\\4\end{bmatrix}}_{c_1}, \underbrace{\begin{bmatrix}-4\\2\end{bmatrix}}_{c_2} \right\}$$

⇐ *Please put scrap work for problem 3 on the page to the left* ⇐.
⇓ *Put neat work to be graded for problem 3 below* ⇓.

a) What vectors x are represented by $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix}2\\14\end{bmatrix}$ and $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix}2\\4\end{bmatrix}$?

$$\underline{X} = [\mathcal{B}][\underline{X}]_{\mathcal{B}} = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}2\\14\end{bmatrix} = \boxed{\begin{matrix}-12\\16\end{matrix}}$$

$$\underline{X} = [\mathcal{C}][\underline{X}]_{\mathcal{C}} = \begin{bmatrix}2 & -4\\4 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = \boxed{\begin{matrix}-12\\16\end{matrix}}$$

They happen to be the same.

b) Find a single tidy formula to find the components $\begin{bmatrix}d\\e\end{bmatrix}$ of a vector x in the basis $\mathcal{B}$ if you are given the components $\begin{bmatrix}f\\g\end{bmatrix}$ of x in the basis $\mathcal{C}$.

$$\underline{X} = \underline{X}$$

$$[\mathcal{B}][\underline{X}]_{\mathcal{B}} = [\mathcal{C}][\underline{X}]_{\mathcal{C}}$$

$$[\underline{X}]_{\mathcal{B}} = \underbrace{[\mathcal{B}]^{-1}[\mathcal{C}]}_{\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}}[\underline{X}]_{\mathcal{C}}$$

Side Calculation of $\mathcal{B}^{-1}$

$$\left[\begin{array}{cc|cc}1 & -1 & 1 & 0\\1 & 1 & 0 & 1\end{array}\right] \sim \left[\begin{array}{cc|cc}1 & -1 & 1 & 0\\0 & 2 & -1 & 1\end{array}\right] \sim \left[\begin{array}{cc|cc}1 & -1 & 1 & 0\\0 & 1 & -1/2 & 1/2\end{array}\right] \sim \left[\begin{array}{cc|cc}1 & 0 & 1/2 & 1/2\\0 & 1 & -1/2 & 1/2\end{array}\right]$$

$$\Rightarrow [\mathcal{B}^{-1}] = \begin{bmatrix}1/2 & 1/2\\-1/2 & 1/2\end{bmatrix}$$

$$\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = [\mathcal{B}]^{-1}[\mathcal{C}] = \frac{1}{2}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}2 & -4\\4 & 2\end{bmatrix}$$

$$\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = \frac{1}{2}\begin{bmatrix}6 & -2\\2 & 6\end{bmatrix} = \begin{bmatrix}3 & -1\\1 & 3\end{bmatrix}$$

$$[\underline{X}]_{\mathcal{B}} = \underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[\underline{X}]_{\mathcal{C}}$$

$$\Rightarrow \boxed{\begin{bmatrix}d\\e\end{bmatrix} = \begin{bmatrix}3 & -1\\1 & 3\end{bmatrix}\begin{bmatrix}f\\g\end{bmatrix}}$$

c) A student claims that the desired formula is $\begin{bmatrix}d\\e\end{bmatrix} = \begin{bmatrix}5 & -2\\5 & 1\end{bmatrix}\begin{bmatrix}f\\g\end{bmatrix}$. Does this formula make the right prediction for the component vector $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix}f\\g\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}$?

$$\begin{bmatrix}d\\e\end{bmatrix} = \begin{bmatrix}5 & -2\\5 & 1\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}$$

$$[\underline{X}]_{\mathcal{B}} = \begin{bmatrix}2\\14\end{bmatrix}$$

Note that in problem (a) we showed that $[\underline{X}]_{\mathcal{B}} = \begin{bmatrix}2\\14\end{bmatrix}$ and $[\underline{X}]_{\mathcal{C}} = \begin{bmatrix}2\\4\end{bmatrix}$ both correspond to the vector $\underline{X} = \begin{bmatrix}-12\\16\end{bmatrix}$.

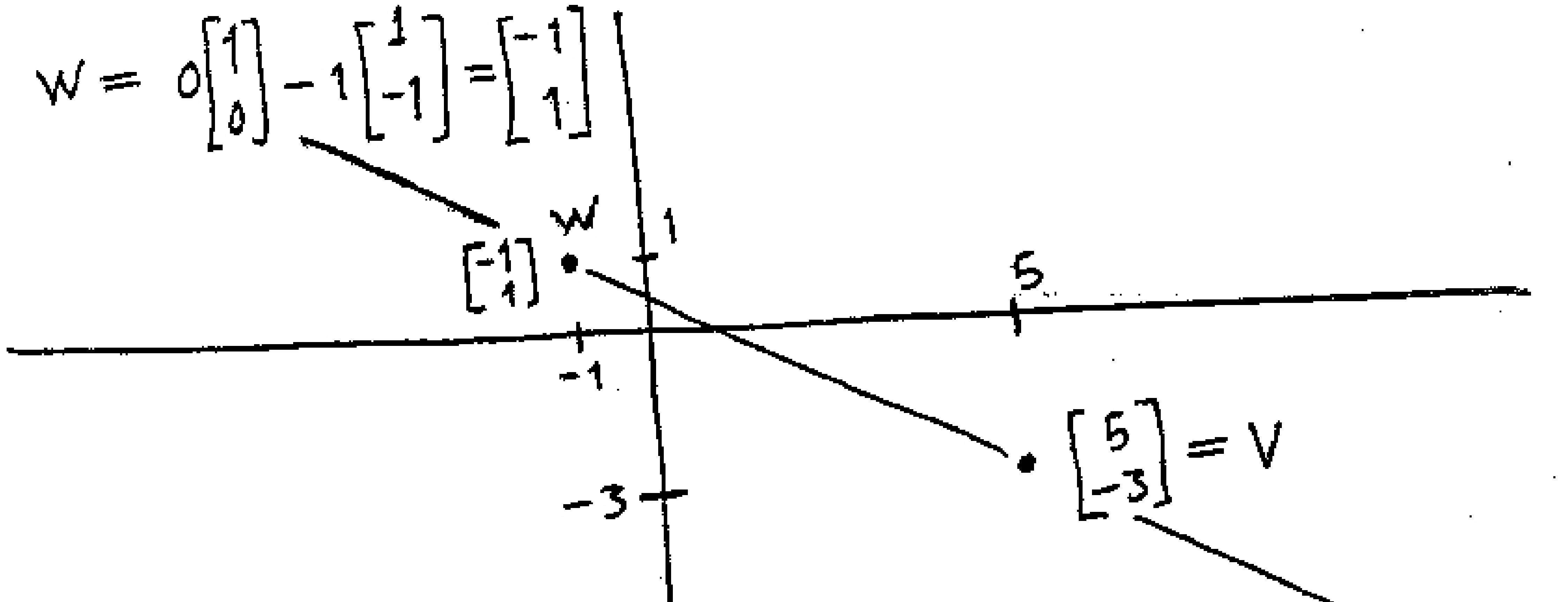
$\Rightarrow$ $\boxed{\text{The prediction is correct in this case.}}$ {Though the formula is wrong!}

M294 P III SP87

1 a) $\mathcal{B}[v] = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ so $v = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{cases} c_1 = 5 \\ c_1 + c_2 = -3 \end{cases}$, $c_2 = -8$ and $\mathcal{B}'[v] = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$

(b) $w = 0\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 1\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



(c) NO. Because the line does not pass through the origin. (Doesn't include $\underline{0}$ vector.)

M294 F SP87

2) $b_1\underline{v}_1 + b_2\underline{v}_2 = b_1'\underline{v}_1' + b_2'\underline{v}_2'$

$\therefore\ b_1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_2\begin{pmatrix} 1 \\ 1 \end{pmatrix} = b_1'\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_2'\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\therefore\ \begin{array}{l} b_1' = b_1 + b_2 \\ b_2' = b_2 \end{array}$

$\therefore\ \begin{pmatrix} b_1' \\ b_2' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \Rightarrow \boxed{{}_{\beta'}[I]_{\beta} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}$

M293 SP96 F #8

6) The answer is a).