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#4. a)

$$\left[\begin{array}{ccc|c} 0 & -4 & 2 & 0 \\ 2 & 6 & 0 & 0 \\ 3 & 7 & 1 & 0 \end{array}\right] \xrightarrow[\text{Row 1/-2}]{\text{Row 2/2}} \left[\begin{array}{ccc|c} 0 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 7 & 1 & 0 \end{array}\right] \xrightarrow{\text{Row 1 + Row 2}} \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 7 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[\text{Row 3 - 3Row 1}]{\text{Row 2 - Row 1}} \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & -8 & -2 & 0 \end{array}\right] \xrightarrow{\text{Row 3 - 4Row 2}} \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \xrightarrow[\text{Row 1 - Row 3}]{\text{Row 3/2, Row 2 + Row 3}}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[\text{Row 1 - 5Row 2}]{\text{Row 2/-2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \qquad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$

SINCE $A\underline{x} = \underline{0}$ HAD ONLY THE TRIVIAL SOLUTION, THEN THE COLUMNS OF MATRIX A ARE LINEARLY INDEPENDENT.

b)

$W = x_1 \underline{v_1} + x_2 \underline{v_2} + x_3 \underline{v_3}$

c) SINCE W SPANS ALL OF $\mathbb{R}^3$, W IS A 3-DIMENSIONAL SPACE

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#5

FOR $\underline{y}$ TO BE IN SPAN $\{\underline{v_1}, \underline{v_2}, \underline{v_3}\}$, IT MUST BE A LINEAR COMBINATION OF $\underline{v_1}, \underline{v_2}, \underline{v_3}$. ANOTHER WAY TO LOOK AT IT IS $A\underline{x} = \underline{b}$ MUST BE CONSISTENT FOR $A = [\underline{v_1}\ \underline{v_2}\ \underline{v_3}]$ AND $\underline{b} = \underline{y}$.

$$\left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{array}\right] \xrightarrow[\text{Row 3 + 2Row 1}]{\text{Row 2 + Row 1}} \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & h-8 \end{array}\right] \xrightarrow{\text{Row 3 - 3Row 2}} \left[\begin{array}{ccc|c} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-8+3 \end{array}\right]$$

$\therefore$ FOR THERE TO BE A SOLUTION $h - 8 + 3 = 0$ OR $\underline{h = 5 \ \ (b)}$

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#6.

D) IF A SET CONTAINS MORE VECTORS THAN THERE ARE ENTRIES IN EACH VECTOR, THEN THE SET IS LINEARLY DEPENDENT.

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7) The answer is False.

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8) FALSE, LET A BE A $m \times n$ MATRIX, WHERE $m=6$ AND $n=4$, THE ONLY WAY FOR THE COLUMNS OF A (4 VECTORS IN $\mathbb{R}^6$) TO SPAN $\mathbb{R}^6$. IS TO HAVE A PIVOT POSITION IN EVERY ROW. THIS IS IMPOSSIBLE WITH 6 ROWS AND 4 COLUMNS. $\therefore$ A SET OF 4 VECTORS IN $\mathbb{R}^6$ CAN NOT SPAN $\mathbb{R}^6$.

(13) M294 FA98 P III #3

a) If A is a (possibly not square) matrix with A^TA invertible are the columns of A linearly independent? (yes, no, maybe).

Assume cols. A not L.I. $\Rightarrow A\underline{x}=\underline{0}$ has non-trivial solns.

Multiply both sides by $A^T \Rightarrow A^TA\underline{x}=A^T\underline{0}$

$\Rightarrow$ (1) $A^TA\,\underline{x}=\underline{0}$ has non-trivial solns.

BUT, prob. statement said A^TA was invertible.

so (1) has no non-triv. solns.

CONTRADICTION $\Rightarrow$ $\boxed{\text{YES, cols. of } A \text{ L.I.}}$