F92 PII MATH 293 #1

6)
$$[A\,|\,b]=\left[\begin{array}{cccc|c}1&2&-2&0&5\\3&1&-2&-1&5\\-1&3&-2&1&5\end{array}\right]\xrightarrow[\text{new } r_3=r_3+r_1]{\text{new } r_2=r_2-3r_1}\left[\begin{array}{cccc|c}1&2&-2&0&5\\0&-5&4&-1&-10\\0&5&4&1&10\end{array}\right]\xrightarrow[\text{new } r_3=r_3+r_2]{\text{new } r_1=r_1+\frac{2}{5}r_2}$$

$$\left[\begin{array}{cccc|c}1&0&-2/5&-2/5&1\\0&-5&4&-1&-10\\0&0&0&0&0\end{array}\right]\xrightarrow{\text{new } r_2=-\frac{1}{5}r_2}\left[\begin{array}{cccc|c}1&0&-2/5&-2/5&1\\0&1&-4/5&1/5&2\\0&0&0&0&0\end{array}\right]$$

$$\begin{cases}x_1-\frac{2}{5}x_3-\frac{2}{5}x_4=1\Rightarrow x_1=1+\frac{2}{5}x_3+\frac{2}{5}x_4\\x_2-\frac{4}{5}x_3+\frac{1}{5}x_4=2\Rightarrow x_2=2+\frac{4}{5}x_3-\frac{1}{5}x_4\\x_3\text{ is free}\\x_4\text{ is free}\end{cases}\Rightarrow\vec{x}=\begin{Bmatrix}x_1\\x_2\\x_3\\x_4\end{Bmatrix}=\begin{Bmatrix}1+\frac{2}{5}x_3+\frac{2}{5}x_4\\2+\frac{4}{5}x_3+\frac{1}{5}x_4\\x_3\\x_4\end{Bmatrix}=\boxed{\begin{Bmatrix}1\\2\\0\\0\end{Bmatrix}+\begin{Bmatrix}\frac{2}{5}\\\frac{4}{5}\\1\\0\end{Bmatrix}x_3+\begin{Bmatrix}\frac{2}{5}\\-\frac{1}{5}\\0\\1\end{Bmatrix}}$$

Ans.

M293 F SP96 #13

11) The answer is C.

12) MATH 293 FINAL SPRING 1996 #27

$$\begin{array}{cc}&2\\3&\begin{bmatrix}x&x\\x&x\\x&x\end{bmatrix}\\&A\end{array}\begin{bmatrix}x\\x\end{bmatrix}$$

FALSE, IN ORDER TO HAVE A NONTRIVIAL SOLUTION TO $A\underline{x}=0$ THERE MUST BE ATLEAST ONE FREE VARIABLE AND WE CAN NOT DETERMINE THAT.

M293F SP96 #28

13) The answer is False.

MATH293 FINAL SPRING 1996 #36

#14

TRUE, THE FACT THAT $\underline{x}_1$ AND $\underline{x}_2$ ARE SOLUTIONS MEANS THAT ANY LINEAR COMBINATION OF THE TWO SOLUTIONS IS ALSO A SOLUTION.

MATH 293 PRELIM 1 FALL 1997 #2

16) a)

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & -2 \\ -1 & 0 & a \\ 2 & -2 & b \end{array}\right] \xrightarrow[\text{ROW4}-2\times\text{ROW1}]{\text{ROW3}+\text{ROW1}} \left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & -2 \\ 0 & -1 & a+5 \\ 0 & 0 & b-10 \end{array}\right] \xrightarrow{\text{ROW3}+\text{ROW2}} \left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & a+3 \\ 0 & 0 & b-10 \end{array}\right]$$

$\Rightarrow$ TO BE CONSISTENT, $\underline{v}_3$ LIES IN SPAN$\{\underline{v}_1, \underline{v}_2\}$, $a+3=0$, $b-10=0$ $\Rightarrow$ $a=-3$, $b=10$

b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ -1 & 0 & a & 0 \\ 2 & -2 & b & 0 \end{array}\right] \xrightarrow[\text{ROW4}-2\times\text{ROW1}]{\text{ROW3}+\text{ROW1}} \left[\begin{array}{ccc|c} 1 & -1 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -1 & a+5 & 0 \\ 0 & 0 & b-10 & 0 \end{array}\right] \xrightarrow{\text{ROW3}+\text{ROW2}} \left[\begin{array}{ccc|c} 1 & -1 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & a+3 & 0 \\ 0 & 0 & b-10 & 0 \end{array}\right]$$

$$\xrightarrow{\text{ROW1}+\text{ROW2}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & a+3 & 0 \\ 0 & 0 & b-10 & 0 \end{array}\right]$$

$\Rightarrow$ THE COLUMNS OF A MATRIX ARE LINEARLY INDEPENDENT IF AND ONLY IF THE EQUATION $A\underline{x}=\underline{0}$ HAS ONLY THE TRIVIAL SOLUTION.
$\therefore$ THE SET $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ IS LINEARLY INDEPENDENT IN $\mathbb{R}^3$ IF $a \neq 3$ OR $b \neq 10$

c) THIS IS ANOTHER WAY TO PHRASE PART (a).

$\therefore$ $a=-3$

$b=10$

SP 98 P II MATH 294 #1

17) a.

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ -2 & 3 & 7 & 0 \end{array}\right] \xrightarrow{\text{new } r_3 = r_3 + 2r_1} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right] \xrightarrow[\text{new } r_3 = r_3 + 3r_2]{\text{new } r_1 = r_1 + 3r_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 - x_3 = 0 \Rightarrow x_1 = x_3 \\ x_2 - x_3 = 0 \Rightarrow x_2 = x_3 \\ x_3 \text{ is free} \end{cases}$$

$$\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x_3 \\ x_3 \\ x_3 \end{Bmatrix} = \boxed{\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} x_3} \leftarrow \text{ans.}$$

b. The solution of $A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$ is the solution of $A\vec{x} = \vec{0}$ plus $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$.

$$\vec{x} = \boxed{\begin{Bmatrix} 0 \\ 1 \\ -3 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} x_3} \leftarrow \text{ans.}$$

c. i) False, the solution must only have the trivial solution for the columns of A to be linearly independent. $[A|\vec{0}]$ has a free variable, so the homogeneous equation has more solutions than the trivial solution.

ii) True, if $\vec{\omega}$ is the solution set of $A\vec{x} = \vec{b}$, then $A\vec{\omega} = \vec{b}$.

$$A\vec{\omega} \overset{?}{=} \vec{b}$$
$$A(\vec{p} + \vec{v}_h) \overset{?}{=} \vec{b} \qquad \leftarrow \vec{\omega} = \vec{p} + \vec{v}_h$$
$$A\vec{p} + A\vec{v}_h \overset{?}{=} \vec{b}$$
$$\vec{b} + \vec{0} \overset{?}{=} \vec{b} \qquad \leftarrow A\vec{p} = \vec{b} \text{ and } A\vec{v}_h = \vec{0}$$
$$\vec{b} = \vec{b} \checkmark$$

M294 P II FA98 #2

20) a) For a given $n \times n$ matrix C and a given n element vector b it is known that $Cx = b$ has more than one solution. Can you tell from this whether or not the columns of C span R^n?

$\boxed{\text{YES you can tell.}}$ The columns of C do not span R^n.

Why? Non-unique soln. $\Rightarrow A\underline{x} = \underline{0}$ has non-triv. soln.
$\Rightarrow$ rank $A < n \Rightarrow$ cols Lin. Dim $\Rightarrow \dim \operatorname{col} A < n$
$\Rightarrow \operatorname{col} A \neq R^n$

e) Does the equation $Ax = b$ have unique solutions for all b in R^5 for the matrix A below. You will get full credit for MATLAB commands which would generate the answer (you must explain how to interpret the output of the commands).

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$$

$\boxed{\text{No, not unique soln.}}$

note: $A \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \operatorname{nul} A \neq 0 \Rightarrow$ non-unique solns.

one of 10 solns.

```
>A = [ 1 2 3 4 5;
       6 7 8 . . .
       .
       .         ];

>A^(-1)
```

If response is an error message $\Rightarrow$ singular matrix $\Rightarrow$ some $\underline{b}$ not in col A.

M294 P II FA98 4f_2

21) f) Would any of your answers above change if you changed A by randomly changing 3 of its entries in the 2nd, third, and fourth columns to different small integers and the corresponding reduced echelon form for B was presented? (yes?, no?, probably?, probably not?, ?)

Since this change is <u>very</u> likely to keep A invertible

$\Rightarrow$ reduced echelon form will still be $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ All answers above will <u>probably</u> not change. The answer to (d) will <u>definitely</u> not change.

MATH 293 PRACTICE PRELIM #1

22) a)

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ -1 & 2 & 5 & 0 & 1 & 0 \\ 2 & 1 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\text{Row3}-2\text{Row1}]{\text{Row2}+\text{Row1}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 5 & 6 & 1 & 1 & 0 \\ 0 & -5 & -5 & -2 & 0 & 1 \end{array}\right] \xrightarrow[\text{Row3}\times -1]{\text{Row2}+\text{Row3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & 5 & 2 & 0 & -1 \end{array}\right] \xrightarrow{\text{Swap Row2 and Row3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 5 & 5 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{\text{Row2}/5}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2/5 & 0 & -1/5 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{\text{Row2}-\text{Row3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 7/5 & -1 & -6/5 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{\text{Row1}-\text{Row3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 7/5 & -1 & -6/5 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{\text{Row1}-3\text{Row2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/5 & 2 & 13/5 \\ 0 & 1 & 0 & 7/5 & -1 & -6/5 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$AA^{-1} = I$

$$\begin{bmatrix} 1 & 3 & 1 \\ -1 & 2 & 5 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} -11/5 & 2 & 13/5 \\ 7/5 & -1 & -6/5 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \checkmark$$

b) $A\underline{x} = \underline{b}$

$\underline{x} = A^{-1}\underline{b}$

$$\underline{x} = \begin{bmatrix} -11/5 & 2 & 13/5 \\ 7/5 & -1 & -6/5 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix} = \begin{bmatrix} -20 \\ 11 \\ -8 \end{bmatrix}$$