Chapter 5

Fourier and Partial Differential Equations

5.1 Fourier

MATH 294 SPRING 1982 FINAL # 5

5.1.1 Consider the function $f(x) = 2x, 0 \leq x \leq 1$.

a) Sketch the odd extension of this function on $-1 \leq x \leq 1$.

b) Expand the function $f(x)$ in a Fourier sine series on $0 \leq x \leq 1$.

MATH 294 SPRING 1983 PRELIM 3 # 2

5.1.2 Find the Fourier sine series for the function $f(x) = x, 0 \leq x \leq \pi$.

MATH 294 SPRING 1983 PRELIM 3 # 4

5.1.3 **a**) Consider $f(x) = x + 1, 0 \leq x \leq 1$. Make an accurate sketch of the function $g(x)$ which is the odd extension of $f(x)$ over the interval $-1 < x \leq 1$.

b) What is the value of the Fourier series for $g(x)$ in part (a) when $x = 0$?

MATH 294 SPRING 1983 PRELIM 3 # 5

5.1.4 **a**) Name one function $f(x)$ that is both even **and** odd over the interval $-1 < x \leq 1$

b) What is the Fourier sine series of the function from part (a) above?

c) What is the Fourier cosine series of the function $f(x) = 1$ for $0 \leq x \leq 7$

MATH 294 SPRING 1983 FINAL # 2

5.1.5 **a**) Find the Fourier series for the function $f(x) = |x|, -2 \leq x \leq 2$

b) What is the value of the series from part (a) at $x = -\frac{1}{2}$?

MATH 294 FALL 1984 FINAL # 4

5.1.6 **a**) Compute the Fourier Cosine series of the function $f(x)$ given for $0 \leq x \leq L$ by

$$f(x) = \begin{cases} 1 & 0 \leq x \leq \frac{L}{2} \\ 0 & \frac{L}{2} \leq x \leq L \end{cases}$$

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MATH 294 FALL 1984 FINAL # 5

5.1.7 **a)** Compute the Fourier Series solution of the problem

$$\frac{d^2y}{dx^2} - 4y = g(x), 0 < x < L$$

if

$$y(0) = y(L) = 0$$

and

$$g(x) = \begin{cases} 1, & 0 \leq x < \frac{L}{2} \\ -1, & -\frac{L}{2} \leq x \leq L \end{cases}$$

.

MATH 294 SPRING 1985 FINAL # 4

5.1.8 **a)** Compute the Fourier Series of the function

$$f(x) = \begin{cases} 0 & \text{for} \quad -\pi \leq x < 0 \\ 2 & \text{for} \quad 0 \leq x \leq \pi \end{cases}$$

on the interval $[-\pi, \pi]$.

b) State, for each x in $[-\pi, \pi]$, the what the Fourier series for f converges

MATH 294 SPRING 1985 FINAL # 13

5.1.9 What is the Fourier series for the function $f(x) = \sin x$ on the interval $[-\pi, \pi]$?

a) $cosx$

b) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \sin nx$

c) $\sin x$

d) none of these.

MATH 294 FALL 1987 PRELIM 2 # 1

5.1.10 Consider the function $f(x) \equiv 1, 0 \leq x \leq 1$.

a) Extend the function $on - 1 \leq x \leq 1$ in such a way that the Fourier series (of the extended function) converges to $\frac{1}{2}$ at $x = 0$ and at $x = 1$.

b) Compute the Fourier series for **your** extension. (Remark: (a) does not have a unique answer, but (b) forces you to make the simplest choice.)

MATH 294 FALL 1987 PRELIM 2 # 2

5.1.11 Consider the function $f(x) = x$ on $0 \leq x \leq 1$. Compute the Fourier series of the **odd extension** of f on $-1 \leq x \leq 1$. To what value does this series converge when $x = 0; x = 1; x = 39.75$ (3 answers are required)?

MATH 294 SPRING 1985 FINAL # 14

5.1.12 What is the Fourier series for the function $f(x) = \sin x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

a) $cosx$

b) $\sum_{n=1}^{\infty} \frac{1}{n\pi} \sin nx$

c) $\sin x$

d) none of these.

MATH 294 SPRING 1985 FINAL # 15

5.1.13 Compute $\int_{-\pi}^{\pi} \cos 2x \cos 3x dx$.

a) 1
b) $\frac{1}{5}$
c) $\frac{1}{6}$
d) 0
e) none of these.

MATH 294 SPRING 1985 FINAL # 16

5.1.14 To what does the Fourier series, of the function $f(x) = x$ on the interval $[-1, 1]$, converge at $x = 1$?

a) -1
b) 0
c) 1
d) none of these.

MATH 294 SPRING 1985 FINAL # 16

5.1.15 To what does the Fourier series, of the function $f(x) = x$ on the interval $[-1, 1]$, converge at $x = 10$?

a) -1
b) 0
c) 1
d) 10
e) none of these.

MATH 294 SPRING 1987 PRELIM 1 # 12

5.1.16 A MuMath (primitive version of MAPLE) command can be used for full credit on one of these (your choice).

a) What is the **Fourier** series for $\sin 6\pi x$ on the interval $-3 \leq x < 3$.
b) What is the Fourier **sine** series for the function $\sin 6\pi x$ on the interval $-3 \leq x < 3$.
c) What is the Fourier **cosine** series for $\sin 6\pi x$ on the interval $-3 \leq x < 3$.
d) Write out the first four non-zero terms of the **Fourier** series for the function below in the interval $(-3 \leq x < 3)$

$$f(x) = \begin{cases} 0 & \text{if} \quad x < 0; \\ 2 & \text{if} \quad x > 0 \end{cases}$$

.

MATH 294 SPRING 1989 PRELIM 2 # 1

5.1.17 Consider the function $f(x) = 1 - x$ defined on $0 \leq x \leq 1$.

a) Sketch the odd extension of $f(x)$ over the interval $-1 \leq x \leq 1$.
b) Find the Fourier sine series for $f(x)$.
c) What does the series converge to on the interval $0 \leq x \leq 1$?

MATH 294 SPRING 1989 PRELIM 2 # 2

5.1.18 Let $f(x)$ is defined for all x,

a) Show that

$$g(x) = \frac{f(x) - f(-x)}{2}$$

is even.

b) Compute $\int_{-\pi}^{\pi} g(x)\sin(x)dx$

MATH 294 FALL 1989 FINAL # 4

5.1.19 Find the Fourier series for the function $f(x) = x^2, -1 \leq x \leq 1$.
Hint: $a_k = \int_{-1}^{1} f(x)\cos(\pi kx)dx, b_k = \int_{-1}^{1} f(x)\sin(\pi kx)dx$.

MATH 294 SPRING 1990 PRELIM 2 # 4

5.1.20 **a)** Find the Fourier series of

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \text{ and } \frac{\pi}{2} < x \leq \pi \\ 1 & 0 \leq x \leq \frac{\pi}{2} \end{cases}$$

b) Find the Fourier series of

$$g(x) = \begin{cases} 0 & \frac{\pi}{2} \leq x < \pi \\ 1 & 0 \leq x \leq \frac{\pi}{2} \end{cases}$$

c) Find the Fourier sine series of $g(x)$ (defined in (b)).

MATH 294 SPRING 1990 PRELIM 3 # 6

5.1.21 Let $f(x) = 1 - x, 0 \leq x \leq 1$.

a) Use the *even* extension of $f(x)$ onto the interval $[-1, 1]$ to get a Fourier cosine series that represents $f(x)$.

b) Sketch the graph of $f(x)$ and its even extension, and on the same graph sketch the 2^{nd} partial sum of the cosine series.

MATH 294 FALL 1990 FINAL # 16

5.1.22 Given the function $f(x) = 1 - x$ on $0 \leq x \leq 1$.

a) Determine its Fourier *sine* series. What value does this series have at $x = 0$?

b) Write down the integral forms for the coefficients a_n and b_n of the *full* Fourier series.

MATH 294 SPRING 1991 PRELIM 1 # 1

5.1.23 Given the function $f(x) = 1 + x$ on $-1 \leq x \leq 1$, determine its Fourier series. To what values does the series converge to at $x = -1, x = 0$, and $x = 1$?

MATH 294 SPRING 1991 PRELIM 1 # 2

5.1.24 Given the function $f(x) = 1$ on $0 \leq x \leq 1$.

a) Determine its Fourier *sine* series. To what values does the series converge to at $x = 0, x = \frac{1}{2}$, and $x = 1$?

b) Determine its Fourier *cosine* series. To what values does the series converge to at $x = 0, x = \frac{1}{2}$, and $x = 1$?

MATH 294 SPRING 1991 FINAL # 7

5.1.25 Given the function $f(x) = 1 - x$ on $0 \le x \le 1$.

a) Determine its Fourier *sine* series. What value does this series have at $x = 0$?

b) Write down the integral forms for the coefficients a_n and b_n of the *full* Fourier series on $0 \le x \le 1$:

$$1 - x = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi\left(x - \frac{1}{2}\right)}{1/2} + b_n \sin \frac{n\pi\left(x - \frac{1}{2}\right)}{1/2}.$$

MATH 294 FALL 1991 PRELIM 1 # 1

5.1.26 Given the function $f(x)$, defined on the interval $(-\pi, \pi)$:

$$f(x) = \begin{cases} 1 + \sin x, & 0 \le x < \pi \\ \sin x, & -\pi \le x < 0 \end{cases}$$

Determine the Fourier Series of its periodic extension.

MATH 294 FALL 1991 PRELIM 1 # 1

5.1.27 Given the function $f(x) = x - \pi$ on $(0, 2\pi)$.
Determine the Fourier Series of its periodic extension. What value does the Fourier Series converge to at $x = 2\pi$?

MATH 294 FALL 1991 PRELIM 1 # 3

5.1.28 Let $f(x)$ be given by

$$f(x) = \begin{cases} 0 & 0 \le x < 1 \\ 1 & 1 \le x \le 2 \end{cases}$$

Determine the Fourier Series for the odd, periodic extension of $f(x)$ (i.e. the Fourier Sine Series).

MATH 294 FALL 1992 FINAL # 7

5.1.29 Find the Fourier cosine series of the function $f(x) = x^2$ on the interval $0 \le x \le 1$.

MATH 294 FALL 1992 FINAL # 7

5.1.30 For each of the following Fourier series representations,

$1 = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x], 0 < x < \pi,$

$x = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx, 0 < x < \pi,$

$x = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos[(2n-1)x], 0 \le x < \pi,$

a) Find the numerical value of the series at $x = -\frac{\pi}{3}, \pi$, and $12\pi + 0.2$ (9 answers required).

b) Find the Fourier series for $|x|, -\pi < x < \pi$. (Think - this is easy !).

MATH 294 SPRING 1985 FINAL # 4

5.1.31 Find the Fourier series of period 2 for

$$f(x) = \begin{cases} 1 & -1 \le x \le 0 \\ 0 & 0 < x \le 1 \end{cases}$$

MATH 294 FALL 1993 FINAL # 1

5.1.32 Each problem has equal weight. Show all work.
Let $f(x) = 1 (0 < x < 2)$. Consider f_e and f_o to be the *even* and *odd* periodic extensions of f having period 4.

a) Find the Fourier Series of f_0.
b) List the values $f_e(x), f_0(x)$ for $x = 1$, and 3. You should have a total of 4 answers to this part.
c) One main idea underlying Fourier series is the "orthogonality" of functions. Give an example of a function g which is orthogonal (over $-2 \le x \le 2$) to x^2 in other words. $\int_{-2}^{2} g(x)x^2 dx = 0$ with g not identically 0.

MATH 294 FALL 1993 PRELIM 1 # 6

5.1.33 Given the function $f(x) = 1 - x$ on $0 \le x \le 1$.

a) Determine its Fourier *sine* series. What value does this series have at $x = 0$?
b) Write down the integral forms for the coefficients a_n and b_n of the *full* Fourier series.

MATH 294 FALL 1994 PRELIM 3 # 3

5.1.34 Find the Fourier series for the period 4 function $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 \le x < 2 \end{cases}$ and state for which values of x the function is equal to its Fourier series.

MATH 294 FALL 1994 PRELIM 3 # 4

5.1.35 a) A certain Fourier series is given by

$$f(x) = \cos 2x + \frac{\cos 4x}{4} + \frac{\cos 6x}{9} + ...$$

i) sketch 1^{st} and 2^{nd} terms of the series.
ii) sketch the sum of the 1^{st} and 2^{nd} terms
iii) sketch $f(x)$ over several periods, noting the period length.

MATH 294 SPRING 1995 PRELIM 3 # 1

5.1.36 Let

$$f(x) = \left\{ \begin{array}{ll} -1, & \text{if } -1 < x < 0 \\ 1 - x, & \text{if } 0 \le x \le 1 \end{array} \right\}$$

a) Graph on the interval $[-5, 5]$ the function $g(x)$ such that
i) $g(x) = f(x)$if $-1 < x \le 1$
ii) $g(2 - x) = g(x)$ if $1 < x \le 3$
iii) $g(x) = g(x + 4)$ for all x.
b) Write an algebraic expression for $g(x)$ like the one for $f(x)$.

MATH 294 FALL 1995 PRELIM 3 # 2

5.1.37 For the function f defined by $f(x)\begin{cases} 0, & \text{if } 0 \le x < \pi \\ 3, & \text{if } \pi \le x < 2\pi \\ f(-x), & \text{for all } x \\ f(x+4\pi), & \text{for all } x \end{cases}$

a) Calculate the Fourier series of f. Write out the first few terms of the series very explicitly.

b) Make a sketch showing the graph of the function to which the series converges on the interval $-8\pi < x < 12\pi$.

MATH 294 FALL 1995 FINAL # 4

5.1.38 For $f(x) = \begin{cases} 1 & \text{if } |x| \le c \\ 0 & c < |x| < \pi \\ f(x+2\pi) & \text{for all } x \end{cases}$

you are given the Fourier series $f(x) = \frac{c}{\pi} + \sum_{n=1}^{\infty} \frac{2\sin nc}{n\pi} \cos nx$. Here $0 < c < \pi$.

a) Verify that the given Fourier coefficients are correct by deriving them.

b) Evaluate f and its series when $c = \pi$ and $x = \frac{\pi}{2}$, and use the result to derive the formula

$$\frac{\pi}{8} = \frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \frac{1}{18} - ....$$

c) Sketch the graph of the function to which the series converges on the interval $[-3\pi, 6\pi]$.

d) Use the series to help you solve

$$\begin{cases} u_t = u_{xx} \\ u_z(0,t) = u_x(\pi,t) = 0 \\ u(x,o) = \begin{cases} 1 \text{ if } 0 < x < c \\ 0 \text{ if } 0 < x < \pi \end{cases} \end{cases}$$

MATH 294 FALL 1996 PRELIM 3 # 2

5.1.39 For each of the following Fourier series expansion:

i) $f_i(x) = x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx, -\pi < x < \pi$

ii) $f_{ii}(x) = 1 = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\left[(2n-1)\frac{\pi}{2}x\right], 0 < x < 2$

iii) $f_{iii}(x) = x = \pi - \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos\left[(2n-1)\frac{x}{2}\right], 0 \le x \le 2\pi$

a) Give the numerical value of the series at $x = -\pi/3, \pi$ and $12.5\pi = 39.3$. (9 answers required)

b) Find the Fourier series for $|x|, -2\pi < x < 2\pi$.

c) Does $\int x dx = \frac{x^2}{2} = 2\sum_{n=1}^{\infty}(-1)^n n^2(\cos nx - 1); -\pi < x < \pi$?
Does $\frac{dx}{dx} = 1 = 2\sum_{n=1}^{\infty} \cos nx; -\pi < x < \pi$?

Give one reason why you answered "yes" or "no" to these questions.

MATH 294 SPRING 1996 FINAL # 5

5.1.40 Let $u(x,y) = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} e^{-ny} \sin(nx)$

You are also given the Fourier Series $x(\pi - x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} \sin(nx)$ for $0 < x < \pi$

True or False (reason not required)

- **i)** $u_{xx} + u_{yy} = 0$
- **ii)** $u(0,y) = 0$
- **iii)** $lim_{y\to\infty} u(x,y) = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} \sin(nx)$
- **iv)** $u_x(0,y) = 0$
- **v)** $u_x(\pi,y) = 0$
- **vi)** $u(\pi,y) = 0$
- **vii)** $\nabla^2 u = 0$
- **viii)** $u(x,0) = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} \sin(nx)$
- **ix)** $u(x,0) = x(\pi - x)$ if $0 < x < \pi$
- **x)** $div(\vec{\nabla} u) = 0$

MATH 294 SPRING 1997 FINAL # 4

5.1.41 Let $f(x) = 1, 0 < x < \pi$

- **a)** Find a Fourier series for the odd (period 2π extension of $f(x)$.
- **b)** Let $U = Span\{\sin x, \sin 2x, \sin 3x, \sin 4x, \sin 5x\}$. Find $\hat{f}$, the best approximation in U for $f(x)$ with respect to the inner product $< f, g >= \int_{-\pi}^{\pi} f(t)g(t)dt$.
- **c)** Find a Fourier series for the even (period 2π) extension for $f(x)$.
- **d)** Let $V = Span\{\cos x, \cos 2x, \cos 3x, \cos 4x, \cos 5x\}$. Find $\hat{f}$, the best approximation in V for $f(x)$ with respect to the same inner product as above.

MATH 294 FALL 1997 FINAL # 5

5.1.42
- **a)** Find the Fourier cosine series for $f(x) = 1 + x$, on $0 \le x \le 1$.
- **b)** Solve the equation $u_{xx} = 2u_t$, subject to the constrains $u_x(0,t) = u_x(1,t) = 0$, and $u(x,0) = 1 + x$, for $0 \le x \le 1$.

MATH 294 SPRING 1998 PRELIM 1 # 1

5.1.43 Let $f(x) = \begin{cases} 1 & -\frac{\pi}{2} < x < 0 \\ 0 & 0 < x < \frac{\pi}{2} \end{cases}$

- **a)** Extend $f(x)$ as a periodic function, with period π. Sketch this function over several periods.
- **b)** Compute the Fourier series for $f(x)$.
- **c)** Write out the first three non-zero terms of the Fourier series.
- **d)** To what values does the Fourier series converge at $x = -\frac{pi}{4}, x = \frac{\pi}{4}$, and $x = \frac{\pi}{2}$?

MATH 294 SPRING 1984 FINAL # 10

5.1.44 Consider the vector space $C-0(-\pi,\pi)$ of continuous functions in the interval $-\pi \leq x \leq \pi$, with inner product $(f,g) = \int_{-\pi}^{\pi} f(x)(g(x))^*$ where * denotes complex conjugation. Consider the following set of functions $b = \{...e^{-2ix}, e^{-ix}, 1, e^{ix}, e^{2ix}, ...\}$.

a) Are they linearly independent? (Hint: Show that they are orthogonal, that is
$(e^{inx}, e^{imx}) = 0$ for $n \neq m$
$(e^{inx}, e^{imx}) \neq 0$ for $n = m$

b) Ignoring the issue of convergence for the moment, let $f(x)$ be in $C_0(-\pi,\pi)$. Express $f(x)$ as a linear combination of the basis B. That is,

$$f = ...a_{-2}e^{-2ix} + a_{-1}e^{-ix} + a_0 + a_1e^{ix} + a_2e^{2ix} + ...$$

find the coefficients $\{a_n\}$ of each of the basis vectors. Use the results from (a).

c) How does this relate to the Fourier series? Are there coefficients $\{a_n\}$ real or complex? What if B is a set of arbitrary orthogonal functions?

MATH 294 SPRING 1996 PRELIM 3 # 2

5.1.45 **a**) Consider the function f defined by

$$f(x) = \begin{cases} 0, & \text{if } -\pi \leq x < 0, \\ 3 & \text{if } 0 \leq x < \pi, \\ f(x+2\pi), & \text{for all } x. \end{cases}$$

Calculate the Fourier series of f. Write out the first few terms of the series explicitly. Make a sketch showing the graph of the function to which the series converges on the interval $-4\pi < x < 4\pi$. To what value does the series converge at $x = 0$?

b) Consider the partial differential equation $u_t + u = 3u_x$ (which is not the heat equation). Assuming the product form $u(x,t) = X(x)T(t)$, find ordinary differential equations satisfied by X and T. (You are not asked to solve them.)

MATH 294 FALL 1992 FINAL # 7

5.1.46 For each of the following Fourier series representations,
$1 = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x], 0 < x < \pi,$
$x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx, -\pi < x < \pi,$
$x = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos[(2n-1)x], 0 \leq x < \pi,$

a) Find the numerical value of the series at $x = -\frac{\pi}{3}, \pi$ and $12\pi + 0.2$ (9 answers required).

b) Find the Fourier series for $|x|, -\pi < x < \pi$. (Think - this is easy!).

MATH 294 FALL 1998 FINAL # 1

5.1.47 Consider the functions $f(x), S(x)$, and $C(x)$ defined below. [Note that $S(x)$ and $C(x)$ can be evaluated for any x even though $f(x)$ is only defined over a finite interval.]
$f(x) = 1$
$S(x) =$ the function to which the Fourier sin series for $f(x)$ converges (using $L = \pi$), and
$C(x) =$ the function to which the Fourier cos series for $f(x)$ converges (using $L = \pi$).

a) Sketch $S(x)$ over the interval $-3\pi \leq x < 3\pi$. (This can be done without finding any terms in the sin series.)

b) Sketch $C(x)$ over the interval $-3\pi \leq x < 3\pi$. (This can be done without finding any terms in the cos series.)

c) Find $S(x)$ explicitly. (This requires some simple integration.)

d) Compute $C(x)$ explicitly. (This can be done with no integration. If done with integration all integrals are trivial.)

MATH 294 SPRING 1999 PRELIM 3 # 3

5.1.48 Consider the function $f(x) = 1, 0 < x < 3$.

a) Calculate the Fourier sine series for $f(x)$ on $0 < x < 3$.

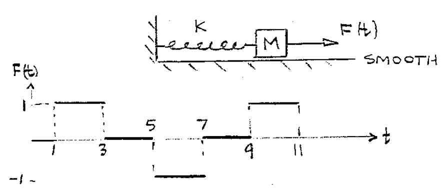
b) Although the function $f(x)$ is defined only over $0 < x < 3$, the Fourier sine series exists for all x. Sketch over $-6 \leq x \leq 6$ the function to which the Fourier sine series for $f(x)$ converges. (Note that you should be able to do this part even if you don't have the correct solution to part a).

MATH 294 SPRING 1983 PRELIM 3 # 1 MAKE-UP

5.1.49 Find the Fourier Cosine Series for the function $f(x) = x, 0 \leq x \leq 1$.

MATH 294 SPRING 1984 FINAL # 11

5.1.50 A simple harmonic oscillator of mass M and stiffness K is acted on by the pulsed periodic force $F(t)$ shown in the figure.



a) Determine the forced response of the oscillator (particular solution) to this excitation – in the form of an infinite series.

First note that the excitation function can be written in the form:

$$F(t) = \frac{4}{\pi} \sum_{i=1}^{n} \frac{\sin \frac{n\pi}{2} \sin \frac{n\pi}{4}}{n} \sin \frac{n\pi t}{4}.$$

Write a brief explanation of this representation.

MATH 294 FALL 1987 FINAL # 4 MAKEUP

5.1.51 Consider the function $f(x) = 3 \; 0 \leq x \leq \pi$

a) Compute the Fourier series of the <u>odd extension</u> of f on $[-\pi, \pi]$.

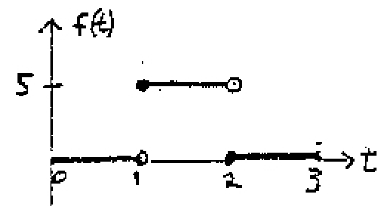
b) To what value does the series (obtained in (a)) converge when $x = 0, x = 1$, and $x = 54$? (3 answers required).

c) Compute the Fourier series of the <u>even extension</u> of f on $[-\pi, \pi]$.

d) To what value does the series (obtained in (c)) converge when $x = 0, x = 1$, and $x = 105, 326$?

MATH 294 SPRING 1987 FINAL # 8 *

5.1.52 It is claimed that the function $f(t)$ graphed below



is equal to the series

$$S(t) = \frac{a_0}{2} + \sum_{i=1}^{n} a_n \cos\left(\frac{n\pi t}{3}\right) + b_n \sin\left(\frac{n\pi t}{3}\right)$$

at all points $0 < t < 3$ except perhaps $t = 1$ and $t = 2$.

a) Extend $f(t)$ any way that you like over the whole interval $-3 < t < 3$ and graph your extension. (There are many answers to this question, and 3 particularly nice ones.)

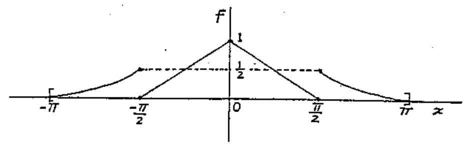
b) For the extension you have drawn, find b_{17}.

c) What is $S(1)$?

d) What is $S(7.75)$?

MATH 294 SPRING 1988 PRELIM 1 # 1

5.1.53 A function $f(x)$ in the interval $(-\pi, \pi)$ is graphed below.



The Fourier series for this function is:

$$\frac{a_0}{2} + \frac{1}{3}\cos(x) + \frac{1}{7}\cos(2x) + a_3\cos(3x) + \ldots + b_1\sin(x) + b_2\sin(2x) + \ldots$$

a) What is the value of $\int_{-\pi}^{\pi} f(x)\cos(2x)dx$? (A number is wanted.)
b) What is the value of the Fourier Series at $x = 0$?
c) What is the value of Fourier Series at $x = \frac{\pi}{2}$?
d) What is your estimate for the value of b_1? (Any well justified answer within .2 of the instructors' best estimate will get full credit.)
e) What is your estimate for the value of $\frac{a_0}{2}$? (Any well justified answer within .2 of the instructors' best estimate will get full credit.)

MATH 294 SUMMER 1990 PRELIM 2 # 6

5.1.54 Let $f(x) = 1 - x, 0 \leq x \leq 1$.
a) Use the *even* extension of $f(x)$ onto the interval $[-1, 1]$ to get a Fourier cosine series that represents $f(x)$.
b) Sketch the graph of $f(x)$ and its even extension, and on the same graph sketch the 2^{nd} partial sum of the cosine series.

MATH 294 FALL 1990 FINAL # 5 MAKEUP

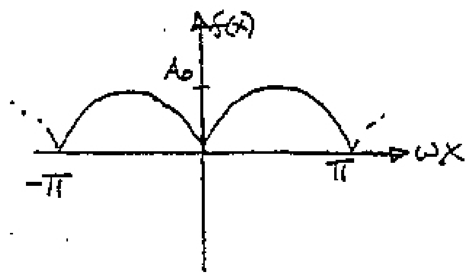
5.1.55 Given the function $f(x) = x - 1$ on $0 \leq x \leq 1$
a) Determine its Fourier <u>cosine</u> series. What value does this series have at $x = 0$?
b) Write down the integral forms for the coefficients a_n and b_n of the <u>full</u> Fourier series.

MATH 294 FALL 1993 PRELIM 3 # 1 *

5.1.56 **a**) Develop a Fourier Series for a <u>rectified</u> sine wave

$$f(x) = \begin{cases} A_0 \sin \omega x & 0 < \omega x < \pi \\ -A_0 \sin \omega x & -\pi < \omega x < 0 \end{cases}$$

$$\text{and } f\left(x + \frac{2\pi}{\omega}\right) = f(x)$$



b) What is the Fourier series for the (unrectified) sine wave: $f(x) = A_0 \sin \omega x$?

c) What is the value of the Fourier series in parts a.) and b.) when evaluated at $x = \frac{3\pi}{2\omega}$?

d) Comment on the derivative of $f(x)$ at $x = 0$. Assuming that the Fourier Series can be differentiated term by term, what is its derivative at $x = 0$?

MATH 294 FALL 1996 PRELIM 3 # 2 MAKE-UP

5.1.57 Let $f(x) = \pi; 0 < x < \pi$.

NOTE: (In parts a and b it is unnecessary to evaluate the integrals for any coefficients a_n or b_n, but the integrals do need to be written explicitly.

a) Express $f(x)$ as a Fourier series of period 2π that involves an infinite series of $\sin\left(\frac{n\pi x}{L}\right)$ terms alone ; $n = 1, 2, 3, \ldots$ Sketch the function to which the Fourier series converges for $-3\pi \leq x \leq 3\pi$.

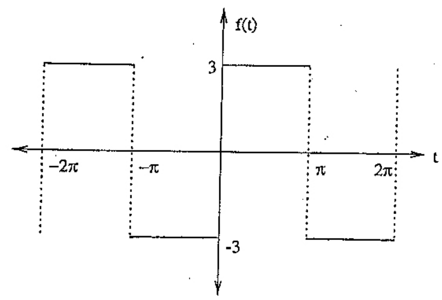
b) Express $f(x)$ as a Fourier series of period 4π that involves an infinite series of $\cos\left(\frac{n\pi x}{L}\right)$ terms alone ; $n = 1, 2, 3, \ldots$ Sketch the function to which the Fourier series converges for $-3\pi \leq x \leq 3\pi$.

c) Sketch an extension of $f(x)$ of period 6π such that the Fourier series of this $f(x)$ contains both sine and cosine terms.

d) Write the simplest possible Fourier series for $f(x)$ (i.e., one containing the fewest terms).

MATH 294 SPRING 1997 PRELIM 1 # 3

5.1.58 Consider the periodic function $f(t)$ shown in the figure below.



a) Find a general explicit expression for the Fourier sine coefficients b_n of $f(t)$

b) Find, explicitly, the first three nonzero terms in the Fourier series for $f(t)$.