2.4 Coordinates

MATH 294 SPRING 1987 PRELIM 3 # 9

2.4.1 For problems (a) - (c) use the bases B and B' below:

$$B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\} \text{ and } B' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}.$$

a) Given that $[\vec{v}]_B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ what is $[\vec{v}]_{B'}$?

b) Using the standard relation between $\Re^2$ and points on the plane make a sketch with the point $\vec{v}$ clearly marked. Also mark the point $\vec{w}$, where $[\vec{w}]_B = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

c) Draw the line defined by the points $\vec{v}$ and $\vec{w}$. Do the points on this line represent a subspace of $\Re^2$?

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2.4.2 A general vector $\vec{v}$ in $\Re^2$ is $\vec{v} = b_1\vec{v}_1 + b_2\vec{v}_2 = b_1'\vec{v}_1{}' + b_2'\vec{v}_2{}'$. where

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \vec{v}_1{}' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{v}_2{}' = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Find a matrix ${}_{B'}[I]_B$ so that $\begin{pmatrix} b_1' \\ b_2' \end{pmatrix} ={}_{B'} [I]_B \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ for all vectors $\vec{v}$ in $\Re^2$.

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2.4.3 **a**) Determine the matrix $H_{E,E}$ which represents reflection of vectors in $\Re^2$ about the y-axis in the standard basis $E = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$. Verify your answer by evaluating the expression

$$H_{E,E} \begin{pmatrix} x \\ y \end{pmatrix}.$$

b) Now consider a basis B which is obtained by rotating each vector of the standard basis by 90 degrees in a counterclockwise direction. Find the change-of-basis matrices $(B : E)$ and $(E : B)$.

c) Find $H_{B,B}$ from the formula $H_{B,B} = (E : B)H_{E,E}(B : E)$.

d) It is claimed that $H_{B,B}$ is equal to the matrix $H_{E,E}$ which represents a reflection about the x-axis in the standard basis. Do you agree? Give geometrical reasons for your answer by drawing a suitable picture.

MATH 293 FALL 1995 PRELIM 3 # 2

2.4.4 Consider the vector space $\Re^3$ and the three bases:

$$\text{the standard basis } E = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\},$$

$$\text{the basis } B = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}, \text{ and}$$

$$\text{the basis } C = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

a) Given the E coordinates of a vector $\vec{x}$, $[\vec{x}]_E = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find $[x]_C$.

b) Given the B coordinates of a vector $\vec{y}$, $[\vec{y}]_B = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, find the coefficients y_j in $\vec{y} = \vec{y}_1\vec{e}_1 + \vec{y}_2\vec{e}_2 + \vec{y}_3\vec{e}_3$.

c) Find the change-of-coordinates matrix ${}_CP_B$ whose columns consist of the C coordinate vectors of the basis vectors of B.

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2.4.5 Let

$$B = \left\{ \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}, C = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

a) Find the change of coordinate matrix from B to C.

b) Find the change of coordinate matrix from C to B.

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2.4.6 Let

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}, C = \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \end{bmatrix} \right\}.$$

Then the change of coordinates matrix from coordinates with respect to the basis C to coordinates with respect to the basis B is

a) $\begin{pmatrix} 2 & -2 \\ 0 & 2 \end{pmatrix}$

b) $\begin{pmatrix} -4 & 4 \\ 0 & -4 \end{pmatrix}$

c) $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

d) $\begin{pmatrix} 0 & 4 \\ 4 & 4 \end{pmatrix}$

e) none of the above

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2.4.7 You are given a vector space V with an inner product $<,>$ and an orthogonal basis $B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3, \vec{b}_4, \vec{b}_5$ for V for which $\left|\left|\vec{b}_i\right|\right| = 2, i = 1, \ldots, 5$. Suppose that $\vec{v}$ is in V and

$$\left\langle \vec{v}, \vec{b}_1 \right\rangle = \left\langle \vec{v}, \vec{b}_2 \right\rangle = 0$$

$$\left\langle \vec{v}, \vec{b}_4 \right\rangle = 3, \left\langle \vec{v}, \vec{b}_4 \right\rangle = 4, \left\langle \vec{v}, \vec{b}_5 \right\rangle = 5$$

Find the coordinates of $\vec{v}$ with respect to the basis B i.e. find c_1, c_2, c_3, c_4, c_5 such that

$$\vec{v} = c_1\vec{b}_1 + c_2\vec{b}_2 + c_3\vec{b}_3 + c_4\vec{b}_4 + c_5\vec{b}_5$$

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2.4.8 Let $T : \wp_1 \to \wp_3$ be defined by

$$T\left[p(t)\right] = t^2 p(t)$$

and take

$$B = \{1, 1+t\}$$

to be the basis of $\wp_3$.

a) Find the matrix of T relative to the bases B and C.

b) Use this matrix to find $T[2+t]$.

c) Let $E = \{1, t\}$ be the standard basis for $\wp_1$. Let $\left[\vec{x}\right]_B$ be the coordinate vector of $\vec{x}$ in $\wp_1$ relative to the basis B, and let $\left[\vec{x}\right]_E$ be the coordinate vector of $\vec{x}$ relative to the basis E. What is the change of coordinate matrix P. such that

$$P\left[\vec{x}\right]_B = \left[\vec{x}\right]_E.$$

[Note: The result of part c) does not depend on the results of parts a) or b)]

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2.4.9 In P^2, Find the change-of-coordinate matrix from the basis

$$B = \left\{1 - 2t + t^2, 3 - 5t, 2t + 3t^2\right\}$$

to the standard basis

$$E = \{1, t, t^2\}.$$

Then write t^2 as a linear combination of the polynomials in B, i.e. give the coordinates of t^2 with respect to the basis B.

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2.4.10 Besides the standard basis ε here are two bases for $\Re^2$:

$$B = \left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{b_1}, \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{b_2} \right\}, C = \left\{ \underbrace{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}_{c_1}, \underbrace{\begin{bmatrix} -4 \\ 4 \end{bmatrix}}_{c_2} \right\}$$

a) What vectors $\vec{x}$ are represented by $[\vec{x}]_B = \begin{bmatrix} 2 \\ 14 \end{bmatrix}$ and $[\vec{x}]_C = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$?

b) Find a single tidy formula to find the components $\begin{bmatrix} d \\ e \end{bmatrix}$ of a vector $\vec{x}$ in the basis B if you are given the components $\begin{bmatrix} f \\ g \end{bmatrix}$ of $\vec{x}$ in the basis C.

c) A student claims that the desired formula is $\begin{bmatrix} d \\ e \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix}$. Does this formula make the right prediction for the component vector $[\vec{x}]_C = \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$?

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2.4.11 Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

a) Find orthogonal eigenvectors $\{\vec{v}_1, \vec{v}_2\}$ of A. [Hint: do not go on to parts d-e below until you have double checked that you have found two orthogonal unit vectors that are eigenvectors of A.]

b) Use the eigenvectors above to diagonalize A.

c) Make a clear sketch that shows the standard basis vectors $\{\vec{e}_1, \vec{e}_2\}$ of $\Re^2$ and the eigenvectors $\{\vec{v}_1, \vec{v}_2\}$ of A

d) Give a geometric interpretation of the change of coordinates matrix, P, that maps coordinates of a vector with respect to the eigen basis to coordinates with respect to the standard basis.

e) Let $\vec{b} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. Using orthogonal projection express $\vec{b}$ in terms of $\{\vec{v}_1, \vec{v}_2\}$ the eigenvectors of A.