1.8 Matrix Inverse

MATH 294 SPRING 1983 PRELIM 1 # 4 294SP83P1Q4.tex

1.8.1 Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

a) Find A^{-1}. Us any method you wish. check your result.

b) Use your inverse above to find all solutions to $A\mathbf{x} = \mathbf{b}$, where

$$\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

MATH 294 SPRING 1983 PRELIM 1 # 5 294SP83P1Q5.tex

1.8.2 **a**) consider the 2 x 2 matrix

$$B = \left[\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right].$$

Find all values of k for which B has no inverse.

b) In the vector space of polynomials of degree two or less are the vectors

$$\{1 - x,\ 2 + x^2,\ x^2 - x - 1\}$$

linearly independent? Give a reason. (You may assume that $\{1,\ x,\ x^2\}$ are linearly independent.)

c) In the vector space S of solutions to the differential equation

$$y" - 3y' + 2y = 0$$

Are the vectors $y = 3e^{-2t}$, $y = e^{-t} - e^{-2t}$, and $y = -e^{-t}$ linearly independent? Give a reason.

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1.8.3 **a)** Find a basis for the vector space of all 2x2 matrices.

b) $\underline{\underline{A}}$ is the matrix given below, $\underline{v}$ is an eigenvector of $\underline{\underline{A}}$. Find any eigenvalue of $\underline{\underline{A}}$.

$$\underline{\underline{A}} = \begin{bmatrix} 3 & 0 & 4 & 2 \\ 8 & 5 & 1 & 3 \\ 4 & 0 & 9 & 8 \\ 2 & 0 & 1 & 6 \end{bmatrix} \text{ with } \underline{v} = [\text{an eigenvector of } \underline{\underline{A}}] = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

c) Find one solution to each system of equations below, if possible. If not possible, explain why not.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \cdot \underline{x} = \underline{b},\ \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

d) Read carefully. Solve for $\mathbf{\underline{x}}$ in the equation $\underline{\underline{A}} \cdot \underline{b} = \mathbf{\underline{x}}$ with:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

e) Find the inverse of the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

MATH 294 FALL 1984 FINAL # 2 294FA84FQ2.tex

1.8.4 **a)** Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

b) Find A^{-1} and A^T.

c) Find the general solution of

$$\frac{d}{dx} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

MATH 294 SPRING 1985 FINAL # 2 294SP85FQ2.tex

1.8.5 For the case $\vec{b} = \vec{0}$, the vector $\vec{x} = \vec{0}$

a) Is always a solution.

b) May or may not be a solution depending on $\underline{\underline{A}}$.

c) Is always the only solution.

d) Is never a solution.

MATH 294 FALL 1985 FINAL # 4 294FA85FQ4.tex

1.8.6 Find an elementary matrix E so that EA is in reduced form, where

$$A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ -1 & 1 & 2 & 3 \\ 3 & 0 & 1 & 0 \end{pmatrix}$$

MATH 294 FALL 1987 FINAL # 3 294FA87FQ3.tex

1.8.7 Either compute the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, or show that no inverse exists.

MATH 294 SUMMER 1989 PRELIM 2 # 5 294SU89P2Q5.tex

1.8.8 Let A be a 2 x 2 matrix with real entries. Assume that A has an inverse Q and $Q = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

a) Find all vectors $\vec{x}$ in $\Re^2$ such that $A\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

b) Find the matrix A.

MATH 294 SPRING 1992 PRELIM 3 # 2 293SP92P3Q2.tex

1.8.9 Here $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

a) Find A^{-1}.

b) Find Xif $AX = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

c) Find $\vec{v}$if $A\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

MATH 293 SPRING 1993 PRELIM 3 # 1 293SP93P3Q1.tex

1.8.10 Given the matrix

$$A = \begin{pmatrix} -2 & 1 & 2 \\ -2 & 2 & 2 \\ -9 & 3 & 7 \end{pmatrix}$$

a) Find det A.

b) Find A^{-1} and check your answer.

MATH 293 SPRING 1993 FINAL # 1 293SP93FQ1.tex

1.8.11 **a)** Find the general solution, and write your answer as a particular solution plus the general solution of the associated homogeneous system.

$$\begin{array}{rcrcrcr} x & - & 5y & + & 4z & = & 3 \\ 2x & - & 3y & + & z & = & -1 \\ -3x & + & y & + & 2z & = & 5 \end{array}$$

b) Check your answer for part a.

c) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -1 & -2 & -2 \end{pmatrix}.$$

d) Check your answer for part c.

MATH 293 SPRING 1994 PRELIM 2 # 6 293SP94P2Q6.tex

1.8.12 **a)** Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

b) Explain what you do to check your result in part (a), and then do it.

c) Compute the determinant of the matrix $A(\lambda) = \begin{pmatrix} 1-\lambda & 1 & 0 \\ 2 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{pmatrix}$, writing your result as a function of λ.

d) Partially check your result by computing the determinant of $A(0)$, and compare this value with the value of the function you found in (c) when $\lambda = 0$.

MATH 293 FALL 1994 PRELIM 2 # 3 293FA94P2Q3.tex

1.8.13 **a)** Find A^{-1} if $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix}$.

b) Use the result of part (a) to solve $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

c) Suppose B and C are N x N invertible matrices and you know B^{-1} and C^{-1}. What is $(BC)^{-1}$ equal to?

MATH 293 FALL 1994 FINAL # 6 293FA94FQ6.tex

1.8.14 The inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is

a) $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 2 \\ 2 & -1 \end{bmatrix}$

c) $\begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} -3 & 2 \\ 2 & 1 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

MATH 293 SPRING 1995 FINAL # 4 293SP95FQ4.tex

1.8.15 Give a formula for

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

when it exists, and prove that your answer is correct.

MATH 293 SPRING 1995 PRELIM 2 # 4 293SP95P2Q4.tex

1.8.16 **a**) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 4 \\ 2 & 1 & 0 \end{bmatrix}.$$

b) Check your result by computing $A^{-1}A$.

c) Use the result to find the solution of

$$\begin{array}{rcrcrcr} x & - & y & + & 2z & = & 5 \\ -x & + & 2y & + & 4z & = & -2 \\ & & 2x & + & y & = & 7 \end{array}$$

MATH 293 FALL 1995 PRELIM 2 # 5 293FA95P2Q5.tex

1.8.17 **a**) Find the inverse, if it exists, of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

b) Verify your answer for part (a).

c) Consider the matrix

$$B = \begin{bmatrix} \lambda & 1 \\ 1 & \lambda \end{bmatrix}$$

where λ is an unspecified parameter. For what values of λ (if any) does B^{-1} not exist?

MATH 293 SPRING 1996 PRELIM 2 # 5 293SP96P2Q5.tex

1.8.18

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \ B = \begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Is A invertible? If yes, explain why and find A^{-1}. If no, explain why?

MATH 293 SPRING 1996 PRELIM 2 # 6 293SP96P2Q6.tex

1.8.19 Let I be the 3-by-3 identity matrix, let 0 be the 3-by-3 matrix consisting of all zeros, let A be the matrix in the question above, and let D be the 6-by-6 matrix given as four 3-by-3 blocks

$$D = \begin{pmatrix} I & A \\ 0 & I \end{pmatrix}.$$

Find D^{-1}.

MATH 293 SPRING 1996 FINAL # 9 293SP96FQ9.tex

1.8.20 Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}.$$

Then the element in the 3rd row, 2nd column of A^{-1} is:

a) -1

b) $-\frac{1}{2}$

c) 1

d) $\frac{1}{2}$

e) none of the above.

MATH 293 SPRING 1996 FINAL # 10 293SP96FQ10.tex

1.8.21 Suppose

$$A^{-1} = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{pmatrix}, \ \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

What is the solution of $A\mathbf{x} = \mathbf{b}$?

a) $\begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

d) $\begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix}$

e) none of the above

MATH 293 FALL 1996 PRELIM 2 # 2 293FA96P2Q2.tex

1.8.22* Matrix algebra. Let $[A]$ be the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and let } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \text{ and let } \vec{c} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

a) Find all solutions $\vec{x}$ to $A\vec{x} = \vec{b}$ and check your answer by substitution.
b) Find all solutions $\vec{x}$ to $A\vec{x} = \vec{c}$ and check your answer by substitution.
c) Give a reason why you believe that A^{-1} does or does not exist.

MATH 294 SPRING 1997 PRELIM 2 # 7 294SP97P2Q7.tex

1.8.23 Let M be the 5-by-5 matrix

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 100 & 0 & & & \\ 200 & 0 & & A & \\ 300 & 0 & & & \end{pmatrix} \text{ where } A^{-1} = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & 0 & 0 \end{pmatrix}.$$

Find the inverse of M.

MATH 294 SPRING 1997 FINAL # 6 294SP97FQ6.tex

1.8.24 Find the inverse of matrix M. A is a general matrix and I is the identity matrix.

$$M = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$$

MATH 294 FALL 1997 PRELIM 1 # 4 294FA97P1Q4.tex

1.8.25 Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -2 & -1 \\ 1 & 1 & 1 \\ -1 & 6 & 4 \end{pmatrix}.$$

MATH 294 FALL 1997 PRELIM 1 # 5 294FA97P1Q5.tex

1.8.26 Find the inverse of the matrix

$$A = \begin{pmatrix} I & B & 0 \\ 0 & I & B \\ 0 & 0 & I \end{pmatrix},$$

where each block is a 3 by 3 matrix, and $I = I_3$ is the 3 by 3 identity matrix. Hint: the inverse is of the form

$$A^{-1} = \begin{pmatrix} I & E_1 & E_2 \\ 0 & I & E_3 \\ 0 & 0 & I \end{pmatrix},$$

for certain $E_1,\ E_2,\ E_3$.

MATH 294 SPRING 1998 PRELIM 2 # 2 294SP98P2Q2.tex

1.8.27 **a**) Find the inverse of the matrix

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 2 \\ 4 & 0 & 3 \end{bmatrix}.$$

b) True or False?

i) If A and B are invertible, then $A^{-1}B^{-1}$ is the inverse of AB.

ii) If A is an invertible n x n matrix, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b}$ in $\Re^n$.

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1.8.28 Determine whether the following matrices are invertible. If they are invertible then find the inverse.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

MATH 294 FALL 1998 PRELIM 2 # 2bc 294FA98P2Q2bc.tex

1.8.29 (b) Is $B = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ the inverse of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$?

(c) Is the matrix $A = \begin{bmatrix} 5 & 3 & 2 \\ 0 & 7 & 1 \\ 5 & 10 & 4 \end{bmatrix}$ invertible? (You need not calculate the inverse if it does exist.)

MATH 293 SPRING ?? PRELIM 2 # 2 293SPxxP2Q2.tex

1.8.30 **a**) If A and B are 4 x 4matrices such that

$$AB = \begin{bmatrix} 2 & 1 & 1 & 0 \\ -1 & 2 & 2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

show that the column space of A is at least three dimensional.

b) Find A^{-1} if $A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

MATH 293 SPRING ?? FINAL # 3 293SPxxFQ3.tex

1.8.31 **a**) Show that $A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 5 & -4 \\ 1 & -4 & 6 \end{pmatrix}$ is nonsingular without finding A^{-1}.

b) Find A^1.

c) Solve $A\vec{x} = \vec{b}$ where $\vec{b} = (1, -2, 1)$ by using part (b).

MATH 293 Unknown PRACTICE PRELIM ? # 2 293UnknownPPQ2.tex

1.8.32 **a**) For what value of a does the matrix $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 1 & a & 0 \end{bmatrix}$ <u>not</u> have an inverse?

b) A n x n matrix C is said to be orthogonal if $C^t = C^{-1}$. Show that either det C = 1 or det C = -1. Hint: $CC^t = I$.

MATH 293 Unknown PRACTICE PRELIM 2 # 4 293UnknownPP2Q4.tex

1.8.33

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 4 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

calculate the inverse of A. Check your answer.

Find the 3 x 3 matrix X if $AX = B$ where A is the 3 x 3 matrix above and

$$B = \begin{bmatrix} 2 & 1 & 7 \\ 3 & -4 & 0 \\ -1 & 2 & 5 \end{bmatrix}$$