1.1 Introduction to Linear Systems and Row Reduction

MATH 294 FALL 1981 PRELIM 1 # 4 294FA81P1Q4.tex

1.1.1 Solve the following systems of linear equations. If there is no solution, show why. If there are infinitely many solutions, give a general expression.

a) $\begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 1 \\ 1 & 4 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$

b) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ -1 & 2 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$

c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ -1 & 2 & -3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

MATH 294 FALL 1982 FINAL # 1 294FA82FQ1.tex

1.1.2 **a)** Find all possible solutions $\vec{x}$ of $B\vec{x} = \vec{c}$, where

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } \vec{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

b) For the system $C\vec{x} = \vec{b}$, where

$$C = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix},$$

determine all vectors $\vec{b}$ for which the system possesses nontrivial solutions $\vec{x}$.

MATH 294 SPRING 1983 PRELIM 1 # 2 294SP83P1Q2.tex

1.1.3 Consider the system

$$\begin{array}{ccccccccc} x & + & y & - & z & + & w & = & 0 \\ x & & & + & 3z & + & w & = & 0 \\ 2x & + & y & + & 2z & + & 2w & = & 0 \\ 3x & + & 2y & + & z & + & 3w & = & 0 \end{array}$$

a) Find all the solutions to this system.

b) Find a basis for the vector space of solutions to the system above. You need not prove this is a basis.

c) What is the dimension of the vector space of solutions above?

d) Is the the vector $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$ a solution to the above system?

MATH 294 SPRING 1984 FINAL # 3 294SP84FQ3.tex

1.1.4 Find the general solution, or else show that the system has no solutions:

$$\begin{array}{rcrcrcrcr} 7x_1 & - & 3x_2 & + & 4x_3 & & & = & -7 \\ 2x_1 & + & x_2 & - & x_3 & + & 4x_4 & = & 6 \\ & & x_2 & & & - & 3x_4 & = & -5 \end{array}$$

MATH 294 FALL 1985 FINAL # 2 294FA85FQ2.tex

1.1.5 Find the general solution of the system

$$\begin{array}{rrcrcrcrcr} & & - & x_2 & + & 3x_3 & + & 2x_4 & = & 1 \\ - & 2x_1 & + & 3x_2 & + & 5x_3 & + & 4x_4 & = & -5 \\ & x_1 & + & x_2 & - & 2x_3 & + & x_4 & = & 8 \end{array}$$

and express your answer in vector form.

MATH 294 FALL 1986 FINAL # 2 294FA86FQ2.tex

1.1.6 **a**) Solve the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 0 & -2 & 4 \\ 2 & 1 & -4 & 6 \\ -1 & 2 & 5 & -3 \\ 3 & 3 & -5 & 4 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 4 \\ 9 \\ 9 \\ 15 \end{bmatrix}.$$

b) Solve the linear system $A\vec{x} = \vec{0}$, where

$$A = \begin{bmatrix} -3 & -1 & 0 & 1 & -2 \\ 1 & 2 & -1 & 0 & 3 \\ 2 & 1 & 1 & -2 & 1 \\ 1 & 5 & 2 & -5 & 4 \end{bmatrix}$$

Express your answer in vector form, and give a basis for the space of solutions.

MATH 294 SPRING 1987 PRELIM 2 # 3 294SP87P2Q3.tex

1.1.7* Find all solutions to:

$$\begin{array}{rrcrcr} & x & + & z & = & 0 \\ - & y & + & 4z & = & 0 \\ & 2y & - & 8z & = & 0 \end{array}$$

MATH 294 FALL 1987 PRELIM 2 # 4 294FA87P2Q4.tex

1.1.8 **a**) Determine the row-reduced form of the matrix:

$$A = \begin{bmatrix} 0 & 2 & 3 & 5 & 0 \\ 0 & 2 & 6 & 8 & -3 \\ 0 & 4 & 6 & 10 & 0 \\ 0 & 4 & 9 & 13 & -4 \end{bmatrix}.$$

b) Find the general solution of $A\vec{u} = \vec{0}$, where

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} \text{ and } \vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

MATH 294 FALL 1987 MAKE-UP 2 # 4 294FA87MU2Q4.tex

1.1.9 Use *row reduction* to either find the solution or show that no solution exists for the system

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & & & = & -2 \\ 3x_1 & + & x_2 & + & 7x_3 & = & -1 \\ 6x_1 & - & 5x_2 & + & 7x_3 & = & 1 \end{array}$$

MATH 294 SPRING 1989 PRELIM 2 # 3 294SP89P2Q3.tex

1.1.10 Consider the system of equations,

$$\begin{array}{rcrcrcr} -x_1 & + & 2x_2 & + & 3x_3 & = & -1 \\ 2x_1 & + & 5x_2 & - & 3x_3 & = & 2 \\ 11x_1 & + & 14x_2 & - & 21x_3 & = & 11 \end{array}$$

a) Find all solutions, if any exist, of the system.

b) Is the set of vectors given by,

$$\begin{bmatrix} -1 \\ 2 \\ 11 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 14 \end{bmatrix}, \text{ and } \begin{bmatrix} 3 \\ -3 \\ -21 \end{bmatrix}$$

linearly independent or dependent?

MATH 294 SUMMER 1989 PRELIM 2 # 1 294SU89P2Q1.tex

1.1.11 **a**) Find all solutions to

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & - & 4x_3 & + & 3x_4 & = & 1 \\ x_1 & + & 2x_2 & - & 2x_3 & + & 2x_4 & = & 1 \\ 2x_1 & + & 4x_2 & - & 2x_3 & + & 3x_4 & = & 2 \end{array}$$

using only the row reduction method.

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1.1.12* Find all the solutions. Write your answers in vector form.

a)
$$\begin{array}{rcrcrcr} 2x_1 & - & 4x_2 & - & 2x_3 & = & 0 \\ 5x_1 & - & x_2 & - & x_3 & = & 6 \\ -3x_1 & + & 2x_2 & + & x_3 & = & -2 \end{array}$$

b)
$$\begin{array}{rcrcrcr} -x_1 & + & 3x_2 & + & 2x_3 & = & 1 \\ 3x_1 & - & 2x_2 & - & x_3 & = & 3 \\ x_1 & + & 4x_2 & + & 3x_3 & = & 5 \end{array}$$

c)
$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & - & 4x_3 & = & 0 \\ 2x_1 & - & x_2 & - & x_3 & = & 0 \\ 3x_1 & - & 4x_2 & + & x_3 & = & 3 \end{array}$$

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1.1.13 Consider $A\vec{x} = \vec{b}$.

$$\text{Where } A\text{is } \begin{pmatrix} 1 & 3 & 5 & -1 \\ -1 & -2 & -5 & 4 \\ 0 & 1 & 1 & -1 \\ 1 & 4 & 6 & -2 \end{pmatrix}.$$

a) Solve for $\vec{x}$ given $\vec{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$.

b) Find a basis for the null space of A.

c) *Without carrying out explicit calculation*, does a solution exist for any $\vec{b}$ in $\mathbf{V}^4$?

MATH 293 FALL 1991 PRELIM 3 # 2 293FA91P3Q2.tex

1.1.14 Solve for the 2 x 2 matrix X if

$$\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} X = \begin{pmatrix} -5 & 1 \\ 0 & 4 \end{pmatrix}.$$

MATH 294 SPRING 1992 PRELIM 3 # 2 293SP92P3Q2.tex

1.1.15 Here $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

a) Find A^{-1}.

b) Find Xif $AX = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

c) Find $\vec{v}$if $A\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

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1.1.16 Find all solutions of the system $A\vec{x} = \vec{b}$ if $A = \begin{bmatrix} 1 & 3 & -2 \\ 3 & -2 & 1 \\ 1 & -19 & 12 \end{bmatrix}$ and

a) $\vec{b} = 0$.

b) $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Express all answers in vector form.

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1.1.17 **a**) Find the eigenvalues and eigenvectors of the matrix

$$B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

b) Let $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$. Find a non singular matrix C such that $C^{-1}AC = D$ where D is a diagonal matrix. Find C^{-1} and D.

c) For which value of a does the system of equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & a \\ 3x & + & 4y & + & 5z & = & 2 \\ -x & & & + & z & = & 0 \end{array}$$

has at least one solution? Explain your answer.

MATH 293 SPRING 1993 FINAL # 1 293SP93FQ1.tex

1.1.18 **a**) Find the general solution, and write your answer as a particular solution plus the general solution of the associated homogeneous system.

$$\begin{array}{rcrcrcr} x & - & 5y & + & 4z & = & 3 \\ 2x & - & 3y & + & z & = & -1 \\ -3x & + & y & + & 2z & = & 5 \end{array}$$

b) Check your answer for part a.

c) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -1 & -2 & -2 \end{pmatrix}.$$

d) Check your answer for part c.

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1.1.19 Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{pmatrix}.$$

a) Find the vectors $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that a solution x of the equation $A\vec{x} = \vec{b}$ exists.

b) Find a basis for the column space $\Re(A)$ of A.

c) It is claimed that $\Re(A)$ is a plane in $\Re^3$. If you agree, find a vector $\vec{n}$ in $\Re^3$ that is normal to this plane. Check you answer.

d) Show that $\vec{n}$ is perpendicular to each of the columns of A. Explain carefully why this is true.

MATH 293 SPRING 1994 PRELIM 2 # 2 293SP94P2Q2.tex

1.1.20 (True/false) The following properties hold for the matrix

$$A = \begin{pmatrix} 2 & -3 & 7 \\ -1 & 4 & 0 \end{pmatrix}:$$

a) If $AM = AN$ then $M = N$, where M and N are 3 x 2 matrices.

b) A has an inverse.

c) A is in reduced row echelon form.

d) A is equal to the matrix $B = \begin{pmatrix} -1 & 4 & 0 \\ 2 & -3 & 7 \end{pmatrix}$.

e) A and B are row equivalent.

f) A and B have the same row reduced form.

g) $(A^T)^T = A$.

h) $B^T A = BA^T$.

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1.1.21* If the reduced row echelon form of

$$\begin{pmatrix} 1 & -1 & 2 & -2 & 3 & -2 & 6 \\ 2 & 0 & 3 & -4 & 1 & -1 & 4 \\ 1 & -3 & -1 & -2 & 2 & -5 & -1 \end{pmatrix} \text{ is } \begin{pmatrix} 1 & 0 & 0 & -2 & -7/4 & -1/2 & -29/8 \\ 0 & 1 & 0 & 0 & -7/4 & 3/2 & -17/8 \\ 0 & 0 & 1 & 0 & 3/2 & 0 & 15/4 \end{pmatrix},$$

then the general solution of the system

$$\begin{aligned} x - y + 2z - 2w &= -2 \\ 2x + 0y + 3z - 4w &= -1 \\ x - 3y - z - 2w &= -5 \end{aligned}$$

is

$$\textbf{a)} \begin{pmatrix} -1/2 \\ 3/2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \textbf{ b) } 1/2\begin{pmatrix} -1 \\ 3 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \textbf{ c) } \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -7/4 \\ -7/4 \\ 3/2 \end{pmatrix},$$

$$\textbf{d)} \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ 0 \\ 0 \end{pmatrix}, \text{ e) } \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \\ -1 \\ -2 \end{pmatrix}.$$

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1.1.22* The reduced row echelon for of $A = \begin{pmatrix} 1 & 0 & -1 & 3 \\ 2 & 2 & 0 & 4 \\ 1 & 4 & 3 & -1 \end{pmatrix}$ is

$$\textbf{a)} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \textbf{ b) } \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \textbf{ c) } \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\textbf{d)} \begin{pmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \textbf{ e) } \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(Note! There is one and only one correct answer.)

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1.1.23 Find the general solution in vector form for the equations

$$\begin{aligned} x + 2y + 2z - w &= 1 \\ 3x + 6y + z + 2w &= 3 \\ -x - 2y + z - 2w &= -1 \end{aligned}$$

MATH 293 FALL 1994 PRELIM 2 # 2 293FA94P2Q2.tex

1.1.24 Use Gauss-Jordan elimination to find all solutions of

$$\begin{array}{rcrcrcl} x & + & 2y & + & 3z & = & b_1 \\ x & + & y & + & z & = & b_2 \\ 5x & + & 7y & + & 9z & = & b_3 \end{array}$$

in the cases that

$$\text{(a)} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \quad \text{and} \quad \text{(b)} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$$

MATH 293 SPRING 1995 PRELIM 2 # 3 293SP95P2Q3.tex

1.1.25 Find the general solution of the wystem of equations

$$\begin{array}{rcrcrcrcl} 2x_1 & & & + & 4x_3 & & & = & 10 \\ 2x_1 & + & x_2 & + & 3x_3 & & & = & 14 \\ 4x_1 & + & x_2 & + & 7x_3 & + & x_4 & = & 27 \\ -2x_1 & + & 2x_2 & - & 6x_3 & + & x_4 & = & 1 \end{array}$$

MATH 293 SPRING 1995 FINAL # 5 293SP95FQ5.tex

1.1.26 Find the general solution of the system of equations

$$\begin{array}{rcrcrcrcr} & & -x_2 & + & 3x_3 & + & 2x_4 & = & 0 \\ -2x_1 & + & 3x_2 & + & 5x_3 & + & 4x_4 & = & -5 \\ x_1 & + & x_2 & - & 2x_3 & + & x_4 & = & 8 \end{array}$$

MATH 293 FALL 1995 PRELIM 2 # 1 293FA95P2Q1.tex

1.1.27* a) Find the general solution of the wystem of equations

$$\begin{array}{rcrcrcrcl} 2x_1 & & & + & 4x_3 & & & = & 10 \\ 2x_1 & + & x_2 & + & 3x_3 & & & = & 14 \\ 4x_1 & + & x_2 & + & 7x_3 & + & x_4 & = & 27 \\ -2x_1 & + & 2x_2 & - & 6x_3 & + & x_4 & = & 1 \end{array}$$

b) Verify your solution.

MATH 293 FALL 1995 FINAL # 3 293FA95FQ3.tex

1.1.28* a) Find the general solution, in vector form, of the equation $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 4 & 4 & 3 \\ 0 & -2 & -4 & -2 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ -2 \end{bmatrix}.$$

Verify your solution.

MATH 293 SPRING 1996 FINAL # 2 293SP96FQ2.tex

1.1.29* Consider the system

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & - & 2x_4 & = & 3 \\ 2x_1 & + & x_2 & + & 3x_3 & + & 2x_4 & = & 5 \\ & & -x_2 & + & x_3 & + & 6x_4 & = & 3 \end{array}$$

A solution of these equations is:

a) The trivial soution.

b) $x_1 = 9, x_2 = 0, x_3 = 0, x_4 = 1$

c) $x_1 = 0, x_2 = 3, x_3 = 0, x_4 = 1$

d) The system has no solution.

e) None of the above.

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1.1.30* Consider the system

$$\begin{array}{rcrcrcl} x & + & & & z & = & 4 \\ 2x & + & y & + & 3z & = & 5 \\ -3x & - & 3y & + & (a^2-5a)z & = & a-8 \end{array}$$

The value of a for which the system has infinitely many solution is:

a) 2

b) 3

c) none

d) 1

e) none of the above.

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1.1.31* **Matrix algebra.** Let $[A]$ be the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and let } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \text{ and let } \vec{c} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

a) Find all solutions $\vec{x}$ to $A\vec{x} = \vec{b}$ and check your answer by substitution.

b) Find all solutions $\vec{x}$ to $A\vec{x} = \vec{c}$ and check your answer by substitution.

c) Give a reason why you believe that A^{-1} does or does not exist.

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1.1.32 Find the general solution of the linear system

$$\begin{array}{rcrcrcrcrcl} & & & & & & x_3 & - & 1 & = & 0 \\ -2 & + & x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & = & 0 \\ & & & & & & x_3 & + & x_4 & = & 1 \end{array}$$

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1.1.33* Let $A = \begin{bmatrix} 9 & 0 & 0 \\ 1 & 0 & -2 \\ 1 & 2 & 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

a) Find the characteristic polynomial $\det(A - \lambda I)$ of A, and find all eigenvalues. ($hint$: $\lambda - 9$ is one factor of the polynomial.)

b) find an eigenvector for each eigenvalue.

c) Write the augmented matrix for the system of equations $A\vec{x} = \vec{b}$ and solve the system by row operations.

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1.1.34 Solve the following system for x_1, x_2, x_3, x_4 and express the general solution in parametric form.

$$\begin{array}{rcrcrcr} 3x_2 & + & x_1 & - & 2x_3 & = & 2 \\ -1 & + & x_2 & + & x_4 & = & 0 \\ & & x_3 & - & x_1 & = & 1 \end{array}$$

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1.1.35 **a**) Consider the problem $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & 2 \\ -1 & 2 & 1 & -3 \end{pmatrix}, \text{and } \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$

Determine the general solution to this problem, in vector form.

b) Find a 2 by 2 matrix B, which is not the zero matrix, with $B^2 = 0$.

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1.1.36 Find the general solution of these equations in vector parametric form.

$$\begin{array}{rcrcr} x_3 & - & x_1 & = & 1 \\ -x_2 & + & x_4 & = & 2 \\ x_2 & - & x_1 & = & 3 \end{array}$$

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1.1.37 **a**) Write the solution set of the system

$$\begin{array}{rcrcrcl} x_1 & - & 3x_2 & - & 2x_3 & = & 0 \\ & & x_2 & - & x_3 & = & 0 \\ -\ 2x_1 & + & 3x_2 & + & 7x_3 & = & 0 \end{array}$$

in parametric form.

b) With

$$A \equiv \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & -1 \\ -2 & 3 & 7 \end{bmatrix},$$

find all solutions to the system

$$A\overrightarrow{x} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}.$$

c) True or False?

i) The columns of A are linearly independent.

ii) The solution set of $A\overrightarrow{x} = \overrightarrow{b}$ is all vectors of the form $\overrightarrow{w} = \overrightarrow{p} + \overrightarrow{v_h}$ where $\overrightarrow{v_h}$ is any solution of $A\overrightarrow{v_h} = \overrightarrow{0}$ and $A\overrightarrow{p} = \overrightarrow{b}$.

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1.1.38* Find all solution to the following matrix equation $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

for each of the following values of $\vec{b}$:

a) $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

c) $\vec{b} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$

MATH 294 FALL 1998 PRELIM 1 # 3 294FA98P1Q3.tex

1.1.39 **a**) Write the following system of equations as (i) a vector equation (ii) as a matrix equation.

$$\begin{array}{ccccccccc} -5x_1 & + & & & x_3 & & & = & 0 \\ 2x_1 & - & x_2 & + & & & 9x_4 & = & 1 \\ 6x_1 & + & 2x_2 & - & 5x_3 & + & x_4 & = & 6 \end{array}$$

b) Find all solutions to the linear system

$$\begin{array}{cccccccc} 2x_1 & + & 4x_2 & 0x_3 & & & = & 4 \\ x_1 & + & 2x_2 & + & x_3 & & = & 1 \\ x_1 & + & 2x_2 & + & x_3 & & = & 0 \end{array}$$

c) Does the above (b) have a solution for any right hand side?

d) Let

$$\vec{u}_1 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 2 \\ -1 \\ -7 \end{bmatrix}, \vec{b} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$$

For what value(s) of h is $\vec{b}$ in the plane spanned by $\{\vec{u}_1, \vec{u}_2\}$?

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1.1.40 Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 3 & 4 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

a) Find *all* $\vec{x}$ for which $C\vec{x} = \vec{b}$, where

$$\vec{b} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$

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1.1.41* The reduced echelon form of the matrix $A = \begin{bmatrix} 3 & 3 & 2 & 3 \\ -2 & 2 & 0 & 2 \\ 1 & 0 & 1 & -2 \\ 0 & -3 & 2 & -1 \end{bmatrix}$ is

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) What is the rank of A.
b) What is the dimenstion of the column space of A?
c) What is the dimension of the null space of A?
d) Find a solution to $A\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.
e) Find the general solution to $A\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.
f) What is the row space of A?
g) Would any of your answers above change if you changed A by randomly changing 3 of its entries in the 2nd, third, and fourth columns to different small integers and the corresponding reduced echelon form for B was presented? (yes?, no?, probaly? probaly not?, ?)

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1.1.42 Consider $A\vec{x} = \vec{b}$ with $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 2 & 5 & 8 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. The augmented matrix of this system is $\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 2 & 3 & 1 \\ -1 & 2 & 5 & 8 & 1 \end{bmatrix}$ which is row equivalent to

$$\begin{bmatrix} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) What are the rank of A and dim nul A? (Justify your answers.)
b) Find bases for col A, row A, and nul A.
c) What is the general solution $\vec{x}$ to $A\vec{x} = \vec{b}$ with the given A and $\vec{b}$?
d) Select another $\vec{b}$ for which the above system has a solution. Give the general solution for that $\vec{b}$.

MATH 293 SPRING ? FINAL # 1 293SPUFQ1.tex

1.1.43* Find the general soliution of the following linear system and express it in vector form

$$\begin{array}{rcrcrcrcr} 2x & - & 3y & + & 0z & - & w & = & -8 \\ -5x & + & 2y & - & 3z & + & 2w & = & -2 \\ 2x & + & 0y & + & 2z & - & w & = & 4 \\ x & - & y & + & z & - & w & = & -2 \end{array}$$

MATH 293 ??? FINAL # 3 293UFQ3.tex

1.1.44* **a**) Give all solutions of the following system in vector form.

$$\begin{array}{rcrcrcr} 6x_1 & + & & & 4x_3 & = & 1 \\ 5x_1 & - & x_2 & + & 5x_3 & = & -1 \\ x_1 & + & & & 3x_3 & = & 2 \end{array}$$

b) What is the null space of the matrix of coefficients of the unknowns in **a)** ?

MATH 294 SPRING 1982 PRELIM 1 # 1 294SP82P1Q1.tex

1.1.45 **a**) Write the system of equations

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ 2x_1 & + & 3x_2 & + & 4x_3 & = & -2 \\ 3x_1 & + & 4x_2 & + & 6x_3 & = & 0 \end{array}$$

in the form $A\vec{x} = \vec{b}$.

b) Find the det A for A in part (a) above.

c) Does A^{-1} exist?

d) Solve the above system of equations for $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

e) Let $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$. Find $A \cdot B$ (i.e. calculate the product AB).

MATH 293 FALL 1991 FINAL # 1 293FA91FQ1.tex

1.1.46 Write the general solution in vector form:

$$\begin{array}{rcrcrcrcr} 2x & - & y & + & z & + & 3w & = & 2 \\ 4x & + & y & - & 3z & + & 5w & = & 6 \\ -x & + & 2y & - & 3z & - & 2w & = & 0 \\ x & + & 4y & - & 7z & + & 0w & = & 4 \end{array}$$

note: This problem is the same as MATH 293 SUMMER 1992 PRELIM 6/30 #2

MATH 293 SPRING 1992 PRELIM 2 # 2 293SP92P2Q2.tex

1.1.47 Find the general solution solution in vector form for the equations

$$\begin{array}{rcrcrcrcr} x & + & 2y & + & 2z & - & w & = & 1 \\ 3x & + & 6y & + & z & + & 2w & = & 3 \\ -x & - & 2y & + & z & - & 2w & = & -1 \end{array}$$

MATH 293 SUMMER 1992 PRELIM 6/30 # 2 293SU92P630Q2.tex

1.1.48 Find the general solution of the equations

$$\begin{array}{rcrcrcrcr} 2x & - & y & + & z & + & 3w & = & 2 \\ 4x & + & y & - & 3z & + & 5w & = & 6 \\ -x & + & 2y & - & 3z & - & 2w & = & 0 \\ x & + & 4y & - & 7z & + & 0w & = & 4 \end{array}$$

MATH 293 SUMMER 1992 FINAL # 1 293SU92FQ1.tex

1.1.49* **a**) Find the general solution, in vector form, of the equations $A\vec{x} = \vec{b}$ where

$$A = \begin{pmatrix} -1 & -3 & 4 & -2 \\ 0 & 2 & 5 & 1 \\ 0 & 1 & -3 & 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$$

b) Solve $AX = B$ where

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

MATH 293 SPRING 1993 PRELIM 2 # 3 293SP93P2Q3.tex

1.1.50 **a**) Solve the linear system:

$$\begin{array}{rcrcrcrcr} 2w & - & 3x & + & y & - & z & = & -1 \\ 4w & + & x & - & 3y & - & z & = & 1 \\ w & + & 2x & - & 3y & - & 2z & = & -2 \\ 2w & - & 3x & - & y & - & 5z & = & -7 \end{array}$$

Write the general solution in vector form as the sum of a particular solution plus the general solution of the associated homogeneous equation.

b) Check your answer, and explain what you do to check.

MATH 294 SPRING 1997 PRELIM 2 # 10 294SP97P2Q10.tex

1.1.51 Two chemicals A and B, are reacting with each other. After one second has elapsed, 90% of chemical A stays chemical A, while 10% turns into chemical B; also, 80% of chemical B stays chemical B, while 20% turns into chemical A. Suppose that the system is in *equilibrium*, i.e. that there is no change in the amount in grams of chemical A or B from one second to the next. If there are 10 grams of chemical A at equilibrium, how many grams of chemical B must there be?

MATH 294 FALL 1997 PRELIM 1 # 6 294FA97P1Q6.tex

1.1.52 A spaceship operator operates daily spaceship service between three planets, $A, B,$ and C. The matrix below shows the traffic during a Monday. The numbers are fractions of the total number of spaceships that start at one location, and go to another destination. For example, the .4 means that 40% of the spaceships that start at C travel to A that day.

$$\begin{array}{cc} from & \begin{array}{ccc} A & B & C \end{array} \\ & \begin{pmatrix} 0 & 1 & .4 \\ .5 & 0 & .6 \\ .5 & 0 & 0 \end{pmatrix} \begin{array}{c} A \\ B \\ C \end{array} \quad to \end{array}$$

The distribution of spaceships at planets $A, B,$ and C on Monday is 10,b, c. Find b and c such that the same distribution of space ships reappears the next day, on Tuesday.

MATH 294 FALL 1998 PRELIM 1 # 4 294FA98P1Q4.tex

1.1.53 Consider the chemical reaction (unbalanced as written below)

$$C_2H_6O + O_2 \rightarrow CO_2 + H_2O.$$

Let $x_1, x_2, x_3,$ and x_4 be the number of molecules of each compound (in the order give above). Find integers $x_1, x_2.x_3, x_4$ that balance this reaction.
Hint: If you order your elements and hence equations as

$$\begin{pmatrix} Oxygen \\ Carbon \\ Hydrogen \end{pmatrix}, \text{or} \begin{pmatrix} O \\ C \\ H \end{pmatrix},$$

you will minimize th number of row operations.

MATH 294 FALL 1998 FINAL # 7 294FA98FQ7.tex

1.1.54 The kingdom of Ferrgrad has three primary industrial sectors: iron, railroad, and coal. Suppose that:

- To produce \$1 of steel, the steel sector consumes \$.2 of steel, \$.1 of railroad, and \$.2 of coal.
- To produce \$1 of railroad transportation that rail sector consumes \$.1 of steel, \$.2 of rail, and \$.4 of coal.
- To produce \$1 of coal, the coal sector consumes \$.2 of steel, \$.2 of rail, and \$.3 of coal.

Ferrograd does not use all its production in the various sectors to maintain the others. Additionally it *exports*

$$\begin{aligned} S_0 &= \$1.2\text{x}10^6 \text{of steel,} \\ R_0 &= \$0.8\text{x}10^6 \text{of railroad transportational services,} \\ C_0 &= \$1.5\text{x}10^6 \text{of coal.} \end{aligned}$$

Define $S, R,$ and C to be the \$ values of annual production of steel, rail, and coal. Set and *do not solve* a matrix equation that will tell you $S, R,$ and C, the values of the annual productions of the three sectors. [Hint: first write three simultaneous equations, one for each output, which relate production to output.]