

Your year in school (semester, degree sought):

Your name:

Cornell
MAE 4735/5735

Final Exam

Dec 10, 2012

No calculators, books or notes allowed.

5 Problems, 150 minutes, no extra time (Cornell rules)

How to get the highest score?

Please do these things:

? If, when working on a problem, you have any *questions* about what you should or should not assume or write, please read these directions again.

↖ • Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.

\vec{v}_{vect} Use correct **vector notation**.

A+ Be (I) neat, (II) clear and (III) well organized.

□ TIDILY REDUCE and box in your answers (Don't leave simplifiable algebraic expressions).

>> Make appropriate `Matlab` code clear and correct.

You can use shortcut notation like " $T_7 = 18$ " instead of, say, " $T(7) = 18$ ".

Small syntax errors will have small penalties.

↗ Clearly **define** any needed dimensions (ℓ, h, d, \dots), coordinates ($x, y, r, \theta \dots$), variables (v, m, t, \dots), base vectors ($\hat{i}, \hat{j}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \dots$) and signs (\pm) with sketches, equations or words.

→ **Justify** your results so a grader can distinguish an informed answer from a guess.

● If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).

≈ Work for **partial credit** (from 60–100%, depending on the problem)

- Put your answer in terms of well defined variables even if you have not substituted in the numerical values.
- Reduce the problem to a clearly defined set of equations to solve.
- Provide Matlab code which would generate the desired answer (and explain the nature of the output).

□ **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem. Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 7: ____/25

Problem 8: ____/25

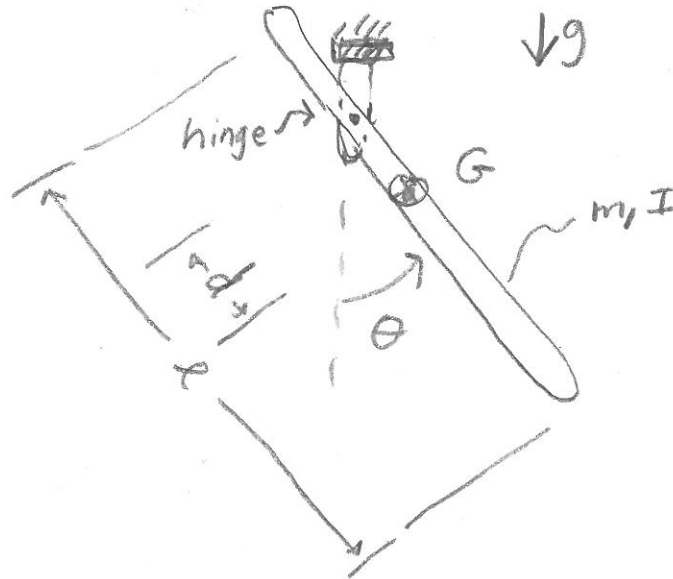
Problem 9: ____/25

Problem 10: ____/25

Problem 11: ____/25

7a) 2D. A stick with length ℓ , mass m and moment of inertia I about its center of mass G is suspended from a hinge on the stick a distance $d < \ell/2$ from G .

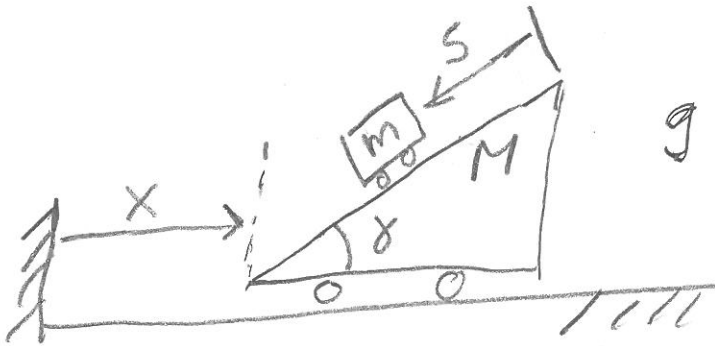
- (i) Find the equations of motion.
- (ii) For given $\theta, \dot{\theta}, m, I, \ell$ and d find the force of the hinge on the pendulum (using any base vectors you like).
- (iii) For given ℓ, m, I and gravitational acceleration g for what d is the period of small oscillation minimized. If you can reduce this to finding the root of a polynomial or transcendental equation, that is good enough, you need not find the root. [Hints: Find the equations of motion \rightarrow Solve them \rightarrow Find the period of small oscillations \rightarrow Minimize.]



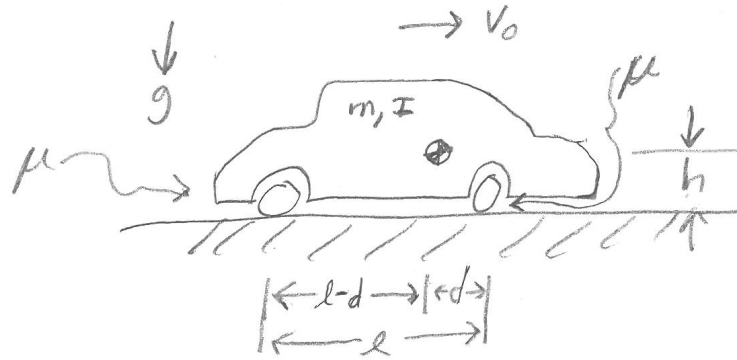
7b) A block with mass M slides without friction on a flat level surface. The top surface has slope γ . A smaller block with mass m slides without friction on the sloped top of the lower block.

- (i) Find two scalar equations from which you could, if you liked, solve for \ddot{s} and \ddot{x} in terms of some or all of $x, \dot{x}, s, \dot{s}, \gamma, m, M$ and g [that is, you need not invert the mass matrix].
- (ii) Some of the statements below are true, some are false, some may be partially true. Say which, and say why (with equations and/or words) clearly enough so that a skeptic would be convinced.

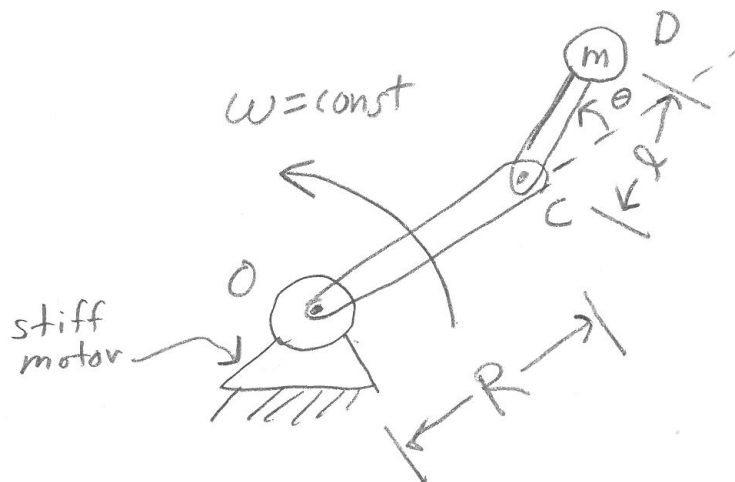
- (a) system potential energy is conserved
- (b) system kinetic energy is conserved
- (c) system total energy is conserved
- (d) system linear momentum is conserved



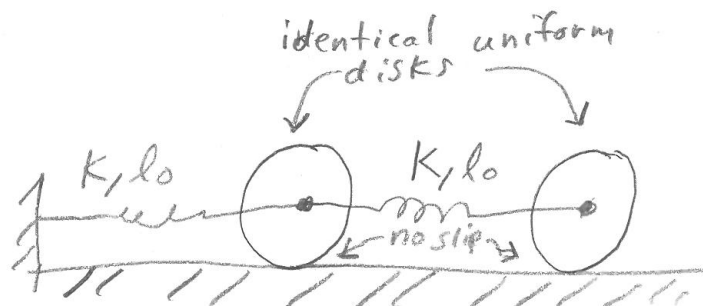
8a) A car with mass m moves at speed v_0 and suddenly slams on the brakes. All four wheels skid with friction coefficient μ (and friction angle ϕ with $\tan \phi = \mu$). Assume the suspension is rigid. In terms of some or all of $d, \ell, h, \mu, \phi, m, I$ and gravity g find how long it takes for the car to come to a stop.



8b) No gravity. One link of a pendulum has radius R and is powered by a stiff motor to rotate at a fixed rate ω . The second link has length ℓ , is massless, and has a point mass m at the end. Find $\ddot{\theta}$ in terms of some or all of $\theta, \dot{\theta}, R, \ell$ and ω .



9) Two uniform round disks with mass m roll without slipping on a flat plane. They are connected to each other and to the left wall with two springs, both with stiffness k and rest length ℓ_0 . Find one normal mode and its corresponding frequency.



10a) This problem concerns the classical 1 DOF spring-mass system with m , k and c .

(a) Make a clear picture of the system.

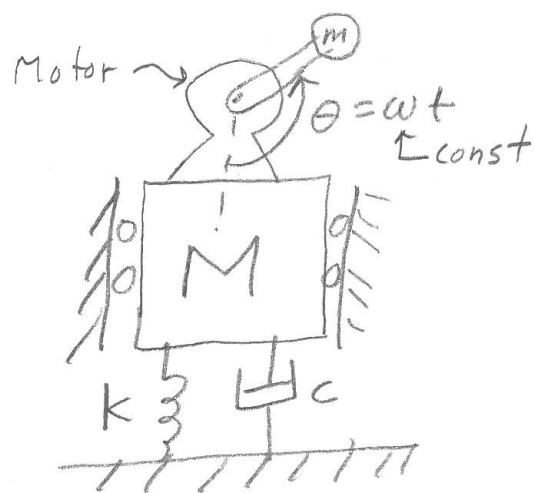
(b) Using any mechanics method you like, find the equations of motion: $m\ddot{x} + c\dot{x} + kx = 0$.

(c) Reduce this to standard form: $\ddot{x} + 2\omega\eta\dot{x} + \omega^2x = 0$

(d) Define both ω and η with equations. Explain the meaning of both terms with words.

(e) Find an equation for the damped natural frequency ω_d .

10b) A platform and motor with total mass M are supported by a spring (k, ℓ_0) and a dashpot (c). The motor spins at constant rate ω so that $\theta = \omega t$. The motor spins an eccentric mass m which is a distance d from the motor shaft. Find the equations of motion for the mass M consisting of the motor and platform (relative to the static equilibrium position).



11) A system has 7 degrees of freedom parameterized by the components of the 7-element vector \vec{x} . The equations of motion, for small motion, are:

$$M\ddot{\vec{x}} + C\dot{\vec{x}} + K\vec{x} = \vec{0}$$

where the matrices M and K are given and symmetric.

- (i) Assume that $C = 0$. Write MATLAB commands to find a constant vector \vec{v} and ω so that

$$\vec{x}(t) = \sin(\omega t)\vec{v}$$

is a solution of the governing equations. Just one normal mode solution is desired.

- (ii) Clearly define at least one non-zero damping matrix C so that, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations of the form:

$$\ddot{r}_i + 2\omega_i \eta_i \dot{r}_i + \omega_i^2 r_i = 0.$$

As for all problems, justify your result.

- (iii) Clearly define the most general damping matrix C for which, with an appropriate change of variables the equations can be re-written as a set of decoupled scalar equations (as written above).

