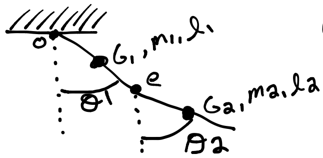
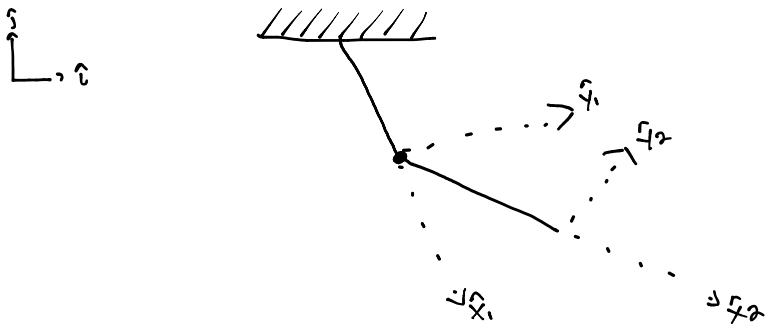
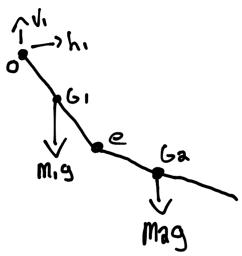
Double Pendulum



θ_1, θ_2 measured from vertical



FBD-1:



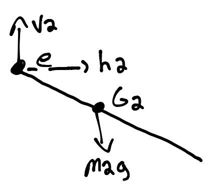
$$\sum \vec{M}_{/o} = \dot{\vec{H}}_{/o}$$

$$\sum \vec{M}_{/o} = (\vec{r}_{G_1/o} \times -m_1 g \hat{j}) + (\vec{r}_{G_2/o} \times -m_2 g \hat{j})$$

$$I \rightarrow \tfrac{1}{12} m l^2$$

$$\dot{\vec{H}}_{/o} = (\vec{r}_{G_1/o} \times m_1 \ddot{\vec{r}}_{G_1/o}) + I_{1/G_1} \ddot{\theta}_1 \hat{k} + (\vec{r}_{G_2/o} \times m_2 \ddot{\vec{r}}_{G_2/o}) + I_{2/G_2} \ddot{\theta}_2 \hat{k}$$

FBD-2:



$$\sum \vec{M}_{/e} = \dot{\vec{H}}_{/e}$$

$$\sum \vec{M}_{/e} = (\vec{r}_{G_2/e} \times -m_2 g \hat{j})$$

$$\dot{\vec{H}}_{/e} = (\vec{r}_{G_2/e} \times m_2 \ddot{\vec{r}}_{G_2/e}) + I_{2/G_2} \ddot{\theta}_2 \hat{k}$$

<u>Expressions for unit vectors</u>: x_1, x_2, y_1, y_2

$$\hat{x}_1 = \sin\theta_1 \hat{i} - \cos\theta_1 \hat{j} \qquad \hat{y}_1 = \cos\theta_1 \hat{i} + \sin\theta_1 \hat{j}$$

$$\hat{x}_2 = \sin\theta_2 \hat{i} - \cos\theta_2 \hat{j} \qquad \hat{y}_2 = \cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}$$

<u>Differentiate Them:</u> unit vectors are constants (no product rules)

$$\dot{\hat{x}}_1 = \frac{d}{dt}(\hat{x}_1) = \frac{d}{dt}(\sin\theta_1 \hat{\imath} - \cos\theta_1 \hat{\jmath}) = \dot{\theta}_1 \cos\theta_1 \hat{\imath} + \dot{\theta}_1 \sin\theta_1 \hat{\jmath} = \dot{\theta}_1 \hat{y}_1$$

<u>Aside:</u> $\dot{q} = \vec{\omega} \times q$, $\dot{\hat{x}}_1 = \dot{\theta}\hat{k} \times \hat{x}_1 = \dot{\theta}\hat{y}_1$

$\dot{\hat{x}}_1 = \dot{\theta}_1 \hat{y}_1$ $\quad \dot{\hat{y}}_1 = -\dot{\theta}_1 \hat{x}_1$ $\quad \dot{\hat{x}}_2 = \dot{\theta}_2 \hat{y}_2$ $\quad \dot{\hat{y}}_2 = -\dot{\theta}_2 \hat{x}_2$

<u>Differentiate Again:</u> cross products this time because we have $\hat{x}, \hat{y}$ and not $\hat{\imath}, \hat{\jmath}$

$$\ddot{\hat{x}}_1 = \ddot{\theta}_1 \hat{y}_1 + \dot{\theta}_1 \dot{\hat{y}}_1 = \ddot{\theta}_1 \hat{y}_1 - \dot{\theta}_1^2 \hat{x}_1$$

$$\ddot{\hat{y}}_1 = -\ddot{\theta}_1 \hat{x}_1 - \dot{\theta}_1^2 \hat{y}_1$$

$$\ddot{\hat{x}}_2 = \ddot{\theta}_2 \hat{y}_2 - \ddot{\theta}_2^2 \hat{x}_2$$

$$\ddot{\hat{y}}_2 = -\ddot{\theta}_2 \hat{x}_2 - \dot{\theta}_2^2 \hat{y}_2$$

$\vec{r}_{G1/O} = \frac{1}{2} l_1 \hat{x}_1$ $\quad \vec{r}_{G2/O} = l_1 \hat{x}_1 + \frac{1}{2} l_2 \hat{x}_2$ $\quad \vec{r}_{G2/e} = \frac{1}{2} l_2 \hat{x}_2$

$\ddot{\vec{r}}_{G1/O} = \frac{1}{2} l_1 \ddot{\hat{x}}$ $\quad \ddot{\vec{r}}_{G2/O} = l_1 \ddot{\hat{x}}_1 + \frac{1}{2} l_2 \ddot{\hat{x}}_2$ $\quad \ddot{\vec{r}}_{G2/e} = \frac{1}{2} l_2 \ddot{\hat{x}}_2$

$I_{1/G_1} = \frac{1}{12} m_1 l_1^2$

$I_{2/G_2} = \frac{1}{12} m_2 l_2^2$

$$\sum \vec{M}_{/O} = -m_1 g (\vec{r}_{G1/O} \times \hat{\jmath}) = -m_1 g \frac{l_1}{2} \sin\theta_1$$

$$-m_2 g (\vec{r}_{G2/O} \times \hat{\jmath}) = -m_2 g (l_1 \hat{x}_1 + \tfrac{1}{2} l_2 \hat{x}_2) \times \hat{\jmath} = -m_2 g l_1 \sin\theta_1 - \tfrac{1}{2} m_2 g l_2 \sin\theta_2$$

Finish This!