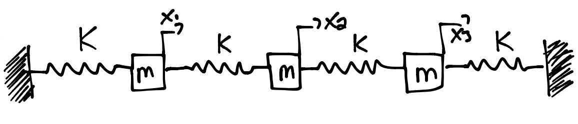
Multi-Dof continued, Vibration Absorption, Normal modes (see diagram from last class)

Guess: $\vec{x} = \underbrace{\bar{x}}_{\text{constant}} e^{i\omega t}$ $\longrightarrow \left[-\omega^2 M + K\right]\bar{x} = \vec{0}$

A complicated solution is the sum of simple solutions

Example:



Can you find the normal modes.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 2K & -K & 0 \\ -K & 2K & -K \\ 0 & -K & 2K \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

guess one of the mode shapes: $\bar{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\rightarrow$ middle mass is stationary left and right mass move opposite each other

$$\omega_1 = \sqrt{\frac{2K}{m}}$$

guess 2: $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ✗ bad! masses cannot move together in the same direction

guess 3: $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ ✗ bad! effective stiffness is not equal for all 3 masses

Variable guess: $\begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}$ Check: is $\frac{K}{m}$ effective for block 1 $=$ $\frac{K}{m}$ effective for block 2

$$\frac{F_1/d_1}{m_1} \overset{?}{=} \frac{F_2/d_2}{m_2} \longrightarrow \frac{(2K-aK)/1}{\cancel{m}} = \frac{2K(a-1)/a}{\cancel{m}}$$

$$(2-a)a = 2(a-1)$$
$$\cancel{2a} - a^2 = \cancel{2a} - 2$$
$$a^2 = 2 \longrightarrow \boxed{a = \pm\sqrt{2}}$$

$\bar{X}_2 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}, \bar{X}_3 = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$ $\omega_1 = \sqrt{\frac{k}{m}(2-\sqrt{2})}, \omega_2 = \sqrt{\frac{k}{m}(2+\sqrt{2})}$

Now add in forcing : $M\ddot{\vec{x}} + K\vec{x} = \bar{F}_0 \sin(\omega t)$

assume $\vec{X}_h(t \to \infty) = \vec{0}$, homogeneous solution decays in time due to damping

μ

We need the particular solution: guess $\vec{x}(t) = \bar{x}\sin(\omega t)$ $\to$ synchronous with the forcing

$M\ddot{\vec{x}} + K\vec{x} = \bar{F}_0 \sin \omega t$

$\underbrace{(-\omega^2 M + K)}_{A}\bar{X} = \bar{F}_0$

Assume $\omega \neq \omega_1 \to$ modal solution

$\bar{x} = (-\omega^2 M + K)^{-1}\bar{F}_0$

Force the system sinusoidally ($\sin\omega t$), eventually it shakes sinusoidally